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NOTES

ON THE

CALIBRATION OF LEVELS

BY

LIEUT. E. A. TANDY, R.E.

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PREFACE.

These notes are based on the careful examination of the tubes of two very fine astronomical levels.

The system of determining the mean value of a division of a level scale, and of applying this mean value to the interpretation of all level corrections, is one which requires a very great perfection of curvature in the tube; as such perfection seems practically unattainable in very delicate levels, the employment of this method has to be safe-guarded by endless precautions; these precautions are very troublesome, and, however closely they may be observed, I think it is generally recognized that the level corrections are very unreliable and unsatisfactory compared with the high standard of accuracy attained on all other points in refined astronomical work.

The object of my experiments was, to obtain a more accurate interpretation of any given level correction, by carefully calibrating the tube. To exhibit the results of this calibration, I have employed a very simple and satisfactory form of diagram, which not only shows all the irregularities in a tube, but automatically allows for them in translating level corrections. The employment of the diagram in practical computation is, if anything, more rapid and convenient than the numerical application of a mean value.

The diagrams have also enabled me to put on foot a few interesting enquiries as to general conditions, and to come to some definite conclusions regarding such questions as the effect of temperature on the curvature of the tube, etc., etc. Even if these conclusions are not entirely new, I think that, considering the many difficulties with which the subject is beset, their independent corroboration from an original point of view may not be without interest.

The calibration described and illustrated in the main body of these notes, is chiefly the result of 4 weeks of experimenting in May 1899, previous to which I had made a large number of preliminary experiments.

The system on which this work was done was sufficiently complete, and the results were most satisfactory, but, as it was my first attempt at a complete calibration, it naturally led to the discovery of many trifling improvements, in the details of procedure, which might be introduced with advantage.

On the completion of the work I had the levels fitted with new scales, and with small cross-levels, etc., etc., and prepared to re-calibrate the tubes, introducing all the additional refinements I had learnt. My duties have prevented my undertaking this re-calibration till quite lately, and other circumstances have necessitated my handing it over, while still in progress, to my successor, Lt. H. McC. Cowie, R.E., who has kindly consented to supervise its completion, and to see the final proof of these notes through the press.

I have embodied in Appendices I and II a full description of the further refinements which have been adopted for the work now in progress. I have also discussed fully, in the main body of the notes, all details which originally hampered and marred my work; also the original calibration, there dealt with, is quite sufficient to completely illustrate my method, and to indicate such claims as it may have to be worthy of further research. This being so, there was little to be gained by indefinitely postponing the publication of these notes, whose chief object is to obtain the collaboration of more experienced observers in the work, and to save them some of the preliminary delays which have hampered me.

The complete examination of levels, as here described, requires considerable time and labour, though the details of procedure are perfectly simple; so that, if it should be generally adopted, it will be convenient to hand over the bulk of the preliminary work to an intelligent assistant, as suggested in Appendices I and II. It is very desirable that any one so employed should have some notion of the theories underlying the process; and it is for this reason that I have, throughout this paper, discussed fully, and in the simplest language, many points which will appear trite and superfluous to the experienced observer, whose patience I must crave in such cases.

The procedure laid down in Appendices I and II has of course no claims to finality. Having fully discussed theories in the notes themselves, I have laid down rules in a somewhat arbitrary manner in the Appendices, simply for the sake of brevity.

DEHRA DŪN,
18th April 1900.

E. A. TANDY.

PART I.—INTRODUCTORY.

Description of Levels.

The two levels which I have examined are of Holmes' best make, and are numbered respectively "6" and "9." They are both furnished with a reservoir, or air-chamber, at one end, from which the length of bubble can be altered at will; the liquid in No. 6 is chloroform, and that in No. 9 spirit, a fact which has added considerably to the interest of my comparisons. They are read by means of vertical brass scales along the top of the tube, having equal divisions, and reading outwards on both sides from 0 in the centre; the lengths of these divisions were arranged by the maker to represent dislevelments of 1″ each, and, as will be seen below, they are fairly near the true mean. The diameter of the tubes is 1 inch and their length about 11 inches, but owing to the space occupied by the reservoirs and the shortness of the scales, etc., the available portion calibrated was only about 6½ inches long.

The following table will show that, as tested by the mean value method, the results from the two levels are good, and would seem to indicate that they are both equally reliable.

No. 6 Level.		No. 9 Level.	
Length of bubble in divisions	Mean value of 1 division	Length of bubble in divisions	Mean value of 1 division
98	0″·900	79	0″·935
96	0 ·931	79	0 ·955
75	0 ·865	70	0 ·950
72	0 ·900	62	0 ·936
69	0 ·904	56	0 ·941
60	0 ·898	42	0 ·956
59	0 ·884	39	0 ·969
50	0 ·888	38	0 ·994
Mean	0″·896	Mean	0″·954

Each of the above figures is the result of running the bubble four times from end to end of the tube on a bubble-tester. The divisions of the scale of No. 6 level are about 0·04 inches and those of No. 9, 0·05 inches in length.

The figures in the above tables are very fairly accordant; but when we come to examine small portions of the tubes separately, great discordances in the mean values at once become apparent. The magnitude of these discordances varies in [different levels; and in the same level there are evident variations due to the position of the bubble in the tube; and, even when in the same mean position, bubbles of different lengths show different results.

As these discordances must seriously vitiate all attempts at interpreting level corrections by the application of a mean value, the discovery of their causes is a matter of great importance.

Discussion of cause of Discordances.

Now in order that a dislevelment of 1″ shall cause a bubble movement of 0·05 inches, it is necessary to grind the interior of the top of the tube to a radius of nearly 300 yards; the delicacy of such an operation in a tube of 7 or 8 inches length will be readily conceived, and, far from expecting absolute accuracy in such a curve, we can only marvel at the degree of approximation actually obtained.

Let us suppose that we have irregularities of the curve as shewn in fig. 1; then it is evident that, if $a\ b$ and $a'\ b'$ represent the two positions of a bubble due to a change of inclination of x'', the distance moved by the bubble will depend on the two curves $a\ a'$ and $b\ b'$; so that if we take the mean distance, measured in divisions, traversed by the two ends of the bubble, (as is always done in applying the "mean value" method) the resultant number of divisions, *viz.*, $\frac{A\,A' + B\,B'}{2}$, will depend on the mean curvature of the arcs $a\,a'$ and $b\,b'$; and, if the movements of the bubble were unaffected by other sources of irregularity, the number of divisions corresponding to x'' dislevelment (*i.e.* the mean value of a division) would vary according to the mean of the two curves traversed by the ends of the bubble.

As, on general grounds, we expect our curve to be imperfect, the above explanation of the discordances found appears to be a reasonable one. I shall speak of this cause as "irregularity of curve"; being permanent, and dependent solely on the position of the bubble, it is obviously susceptible of calibration. Other causes which one might expect to find and which would also account for discordant results are :—

(1). Stickiness in the bubble* causing it to take up an indeterminate position, due to its last direction of motion, or to minute variations in the surface friction in the tube.

(2). Irregular variations, due to the testing instrument, or to disturbances of the pillar on which it stands by earth vibrations.

(3). Effects of slight variations in temperature, which may cause the liquid to be occasionally warmer at one end than the other, and so disturb the nicety of its balance, or which may disturb the levelment of the tester.

(4). Large and persistent differences of temperature, which may disturb the curves of the tube, or otherwise exercise some malign effect.

The first three of these causes are probably quite intangible, though they may be considerable; the fourth might be discoverable if regular, but, on the whole, it was evident that these four causes might very possibly be so considerable as to entirely swamp and conceal all indications of the "irregularity of the curve". As, however, I felt sure of the existence of this latter, I devised a system of recording my results on a diagram, which would *completely* satisfy all the discordances provided that "irregularity of curve" was their *sole* cause, and which would show a marked improvement on the mean value method if irregularity of curve was their *main* cause.

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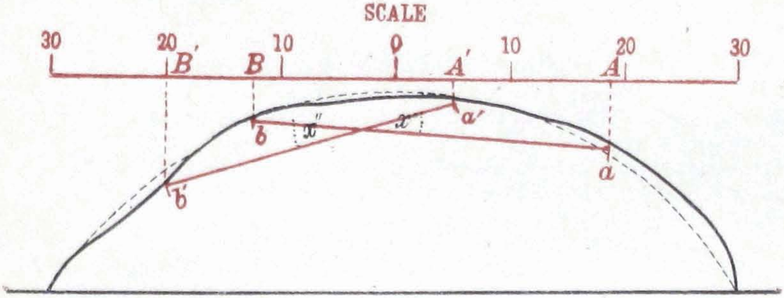
* By this is meant of course stickiness of the liquid, or of its capillary edges bounding the bubble.

Fig. 1



General results obtained from Calibration.

My experiments fully established the feasibility of calibration, and the investigation showed generally that:—

(1). The discordances found in testing levels are chiefly due to "irregularity of curve" as above explained; and that the results when diagrammatically recorded become accordant and satisfactory, except for trifling residual variations, arising either from stickiness of the liquid, instrumental disturbances, or other causes too slight to be dealt with. These residual discrepancies are in no way comparable with the very considerable ones which the diagrammatic method eliminates.

(2). Large and persistent differences of temperature, within a range of 60° to 100° (Fah.), have no appreciable effect.*

(3). Such tests as I have been able to make seem to show that the levels remain appreciably unaltered through a period of six months; time must of course elapse before their constancy over longer periods can be established.

(4). That, in my levels, bubbles of less than 3 inches length become indeterminate, and should not be used, (when under 2½ inches they are very bad). It would appear that the more delicate a level is, the longer must the bubble be; for it is well known that quite small bubbles give satisfactory results in levels whose radius of curvature is small.

(5). That a bubble must be given a sensible momentum to enable it to occupy its new position with precision; if it is attempted to move it 1 division by a slow adjustment, friction may prevent it from attaining its true position of equilibrium, and the result may be misleading. (With my levels, a fairly rapid movement through about a quarter of an inch is necessary to ensure reliability).†

Advantages Claimed for Diagrams.

A level correction can be more quickly obtained from the diagrams than from the method of computation by a mean value; there is therefore no objection to them on the grounds of inconvenience. As the essence of the system is that it automatically allows for all variations from the mean value discovered in testing, an increase of accuracy is inevitable; the amount of this improvement will be proportional to the irregularity of the curve and the quality of the calibration; this fact will be better appreciated when I come to discuss the diagrams in detail, and I will here content myself with stating it. It will also be apparent from the diagrams that large level corrections can be employed with impunity, and that if the diagram is a good one they will be as little liable to error as small ones. The diagrams also eliminate all danger of accumulative errors, which seem very liable to occur with the mean value method; this fact is of immense importance, and the gain in accuracy of single observations is a small matter compared with it. In addition to these, there is the following advantage which nothing but adequate calibration can afford, *viz.*—*from the diagram the irregularities at all portions of the curve can be seen at a glance; this enables us to discriminate between different levels and choose the best; and, further, having got a level, we can mark the unreliable parts of the curve and avoid them during actual work, thus escaping all liability to gross errors.* Finally it will be evident that until a system of calibration is developed, no progress can be made towards the determination of the conditions which cause all the discrepancies in the indications of levels.

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* A rise of temperature of course shortens the bubble, but what I here mean is, that, if, by means of the reservoir, we adjust the bubble to a length of say 3 inches, it will move in a certain way, quite independent of whether the temperature happens to be 60° or 100° F.

† This is a point on which I do not wish to appear too positive; also I have not here discussed the effect of the last direction of motion of a bubble in affecting the position at which it ultimately settles, as I am not yet in a position to make definite statements on the subject. The question is complicated by the fact that a bubble oscillates before settling, so that its last appreciable oscillation may either be in the same direction or opposite to the direction of motion given to it in changing from one position to another: still I have good reason to believe that there is often a very marked effect of this kind, and that the matter is worthy of a careful investigation, which could be easily carried out with a diagram.

Description of Tester.

The bubble tester which I used is known as Cooke's bubble-tester No. 1; it consists of a brass bar nearly 2 feet long supported at one end by a large micrometer screw, and at the other by two solid feet about 4 inches apart and at right angles to the axis of the bar; so that on turning the micrometer the bar revolves about the line joining the two feet. Under the bar is a heavy slab having at one end two steel buttons on which the above-mentioned feet rest, and at the other a coarse brass screw on the flat head of which the foot of the micrometer rests; (this screw enables rough adjustments of the bar to be effected more conveniently than could be done with the micrometer).

The bar is fitted with two sets of vertical **V**'s which enable it to carry two levels at once; the **V** at the opposite end to the micrometer is in each case capable of being raised on the bar by a very delicate adjusting screw; this makes it possible to set the bubble of each level in any required position without moving the micrometer. There is an arrangement by which the weight of the hand in turning the micrometer is taken by an exterior case so that it may not disturb the adjustment of the bar. The head of the micrometer is about 5 inches in diameter and is divided into 100 divisions which are supposed to give 1″ of dislevelment each; the foot of the micrometer screw is semi-spherical, and the flat-headed screw which supports it has a conical recess in which this foot rests; I do not like this last detail, as the slightest unevenness in the semi-spherical point or in the conical recess will have the effect of a periodic error in every revolution of the screw. Otherwise the design of the tester seems simple and satisfactory.

Disturbances which affect the work.

Although the whole was mounted on an isolated pillar, it is difficult to say to what extent the resultant discrepancies of my work may have been due to this tester, owing to the innumerable disturbances to which any such delicate work is liable.

To begin with, it must be borne in mind that a disturbance which raises one end of a level one ten-thousandth part of an inch will move the bubble more than 2″ out of position, and 2″ is a very large quantity compared to the refinement at which I am aiming. Further I have seen during my experiments a bubble run 5″ or 6″ out of position owing to a small "ekka" (or pony cart) passing along a road 20 yards away.[*] It is certain also that minute earth pulsations are always occurring everywhere; when considerable these would generally be fairly slow and regular in their effect, but there is always a liability to very perceptible disturbances from this cause.[†]

Besides these there are numerous less simple disturbances, such as those due to small local temperature variations, which may not only react on the tester, but may disturb the equilibrium of the liquid in the level by causing differences of temperture in the two ends of the tube.

Considering all these things, it will be evident how very liable to fluctuations even perfect levels on perfect testers would be, and how cautious we must be in attributing any occasional discrepancies to indeterminateness and unreliability in the levels. My own impression is that the subject only requires more thorough investigation to show that, as long as bad parts of the curve are avoided, a good level is an instrument of marvellous delicacy and refinement.

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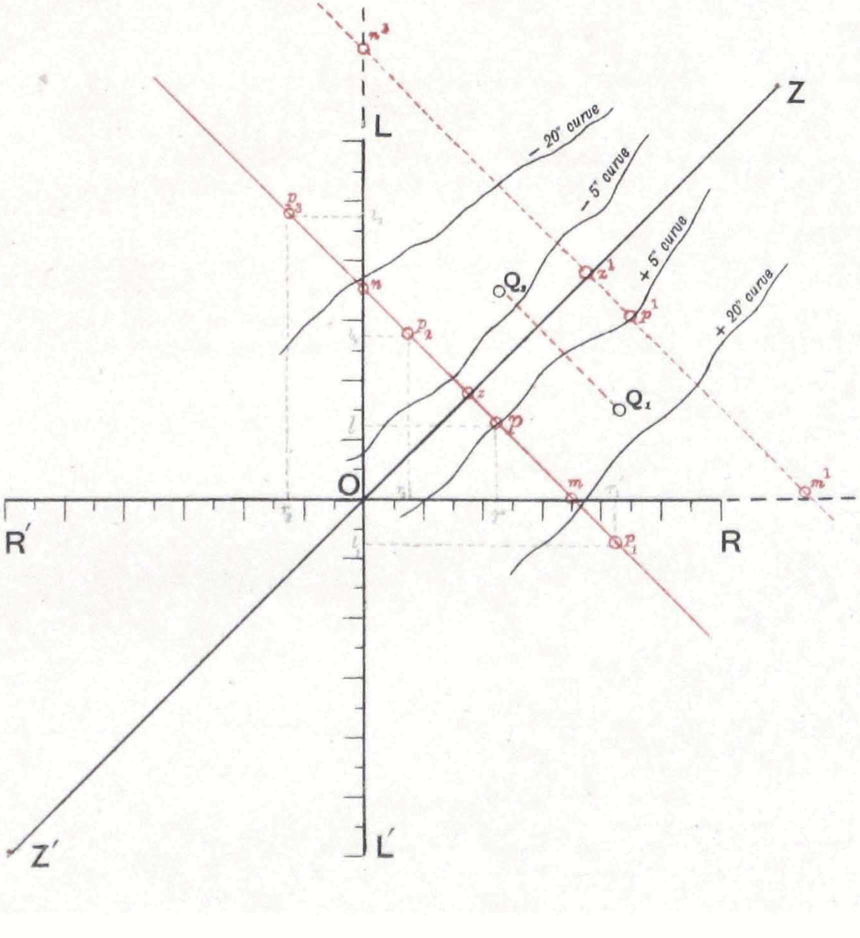
[*] I noticed this several times in No. 6 level which showed far greater sensitiveness to such disturbances than No. 9; after the passage of the ekka the bubble would generally regain its old position fairly accurately; strange to say I never noticed any result from elephants or *quiet* carts passing along the road, though their weight was much greater; it would seem as if the rattle and vibration of a noisy vehicle were necessary to cause the load to take effect. Another curious point was that the *maximum* effect was usually produced on the level some distance before the vehicle actually passed across the line of the axis of the level, and the effect passed off almost immediately after the passage of the vehicle. The effect of my own movements in the vicinity of the instrument was never perceptible.

[†] Vide Milne's "Earthquakes," 4th Edition, Chapter XIX and XX "Earth Tremors" and "Earth Pulsations" especially a case given on page 335 where a bubble moved 10″ in 40 minutes; see also the occasional large and rapid pulse-like surging of undisturbed levels, described on page 336. I have myself seen a bubble suddenly walk off 30″ during my experiments, for no apparent cause, necessitating the abandonment of the series I happened to be working at.

Fig. 2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− 5° curve
+ 5° curve
+ 20° curve
p₃
n
Q₂
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p¹
p₂
z
Q₁
p
O
m
m¹
R′
R
p₁
Z′
L′

I made a considerable number of very careful observations to examine the reliability of my tester, but the results were inconclusive, and I could only infer that the regularity of its micrometer was greater than that of any of the other conditions involved in my work. A careful comparison of its mean value with that of a well-known micrometer on an astronomical instrument shows the error of the mean value of 1 division to be within $0''\cdot004$; this error could hardly amount to $0''\cdot1$ in the largest level correction ever used, and is unnoticeable compared to those with which we are attempting to deal; so I have taken the value of 1 division of the tester to be equal to $1''$ exactly.

PART II.—THE DIAGRAMS.

General Description.

Having sufficiently dwelt on the complex considerations involved in level-testing, and the advantages which a diagrammatic method secures, I shall now explain the particular system of calibration which I adopted.

In computing a level correction the problem is entirely a differential one; that is to say, we are always given two positions of a bubble, and we want to know the amount of dislevelment to which the difference between those two positions corresponds; so that we are free to assume any particular position of a bubble as the zero position, and to calibrate from that. For if our diagram is able to give us the arcs contained between every possible position of a bubble and its zero position, it is evident that, given any two positions, we can at once compute the arc contained between them; for let the arc contained between the first position and zero be x'', and that between the second position and zero be y'', then the arc contained between the two positions, which is the required dislevelment, will be $(x - y)''$.

I have chosen for the zero of any given bubble, (*i.e.*, one of any given length),[*] that position in which its two ends are equidistant from the centre of the scale, *i.e.*, in which the bubble is exactly centrally situated with regard to the scale. This position I shall in future, for brevity, refer to as "equal readings".[†]

Let us take any point O as origin, and two axes R′OR and LOL′, as shewn in fig. 2. Along R′OR mark divisions proportional to the divisions on the bubble scale, and figured similarly, making the central zero point at O; then it is evident that any reading of the right hand end of the bubble may be marked off on the line R′OR; (as the right end of the bubble is generally to the right of the centre of the scale, nearly all actual readings will be on the portion OR and will be positive; when the *whole* of the bubble gets to the left of O, we have a minus reading for the right end which would be plotted on OR′). Similarly, if we mark off the divisions of the scale on the line LOL′, any left hand reading may be plotted on it, and will be positive except when the whole of the bubble is to right of O when it becomes negative and is plotted on OL′.

So that, if the whole sheet of paper be taken to represent all the possible positions of the bubble, and r and l are the readings of the two ends for any given position, then the point p represents on the diagram that particular position of the bubble and no other. Let the total length of the bubble in this case be "B" divisions, then $r + l = \mathrm{B}$; and if $r_1\ l_1$, $r_2\ l_2$, &c., be various readings of this same bubble of length B when in other positions in the tube, then $r_1 + l_1 = \mathrm{B}$, $r_2 + l_2 = \mathrm{B}$

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[*] Throughout this description when speaking of a "given bubble" or the "same bubble" I mean one of a "given length" or of the "same length"; similarly when referring to "different bubbles" I mean such as differ from one another in length. *The terms "bubble" and "level" are never used synonymously.*

[†] We may remind the reader that the scales are marked O in the centre and read outwards both ways, so that under the above assumption the readings at the two ends are equal when the bubble is at zero.

and so on. From this it is evident that the locus of the points representing all possible positions of a bubble of length B will be a straight line cutting the axes at m and n, and having $om = on = B$. I shall in future refer to such a line as a "*bubble-length*," meaning the line on which all the points of a certain lengthed bubble lie. As we have decided that the point of equal readings is to be the zero of any bubble, it is evident that the zero of the bubble "B" is at z or on a line bisecting the angle LOR. Similarly if we take any other bubble of length B′ whose "bubble-length" is $m' n'$, its zero is at z' and also on this line; so that ZOZ′ is the locus of the zeros of all the bubbles.

Now to calibrate our level, let us make the bubble B divisions long and set the tube so that the bubble is at "equal readings," *i.e.*, the right hand end reads $+ \frac{B}{2}$ divisions and the left hand the same; this gives us the point z on the diagram. If we now raise the right hand end of the tube exactly 5″, and read the two ends of the bubble in its new position, we shall, by plotting these two readings r and l on the diagram, get a point p, and shall know in future that the movement zp of this bubble B corresponds to a dislevelment of 5″. Similarly with a bubble of B′ divisions in length, and having its zero at z', we may get a point p', such that $z'p'$ corresponds to 5″; and by doing this with a sufficient number of bubbles we shall get the locus of the curve pp', which will show us the movement of every possible bubble corresponding to an arc of 5″ to the right of its zero, *i.e.* of the + 5″ curve. In the same way we may find the curves corresponding to every second of dislevelment for every possible length of bubble, thus covering our diagram with a series of curves, which, like the four I have drawn in fig. 2, will be all more or less parallel to the zero line OZ in general direction. It is convenient to number the curves outwards from zero in both directions, and to give a positive sign to those which represent the bubble as being to the right of its central position and a negative sign when the bubble is to the left. This convention should be carefully noted, and must not be confused with the perfectly distinct question of the readings of the level-scales, which are only accorded a minus sign in the case of the left end of the bubble being to the right of 0, or the right end to the left of 0, the reason being obvious.

If then we require the true arc represented by a movement of a bubble between any two given positions, we have only to find the two points on the diagram corresponding to the two positions, and the difference between their distances from zero, as shewn in seconds of arc by the curves, will give the required arc; for if Q_1 Q_2 be the two points, then if Q_1 is on the curve + 15″ and Q_2 is at − 9″·3 (decimals of a second would of course be judged by interpolation between the curves by eye), the difference is 15 + 9·3 = 24″·3, which is the arc required.

Discussion of Imaginary Cases.

The above shows the general principle of the diagram without any consideration of practical difficulties in its execution; and, before passing on to these, it will be useful to consider one or two imaginary diagrams, so that we may be the better able to interpret such indications of irregularities in the tube as our real diagrams may give us.

To begin with, it is evident that if we have an immaculate level whose tube has a perfectly regular curve, a movement of 10 divisions, say, would always correspond to a given dislevelment, which, if 1 division was exactly equal to 1″, would be 10″; so that, in this case, perfect calibration would give a diagram of straight lines, as shewn in fig. 3, which is a skeleton diagram, giving only the 5″ and 10″ curves instead of all the single seconds. It may also be noted that I have adopted the convention of enclosing the figures representing right hand readings in circles; the usefulness of this in avoiding confusion will be more fully realised later.

Fig. 3

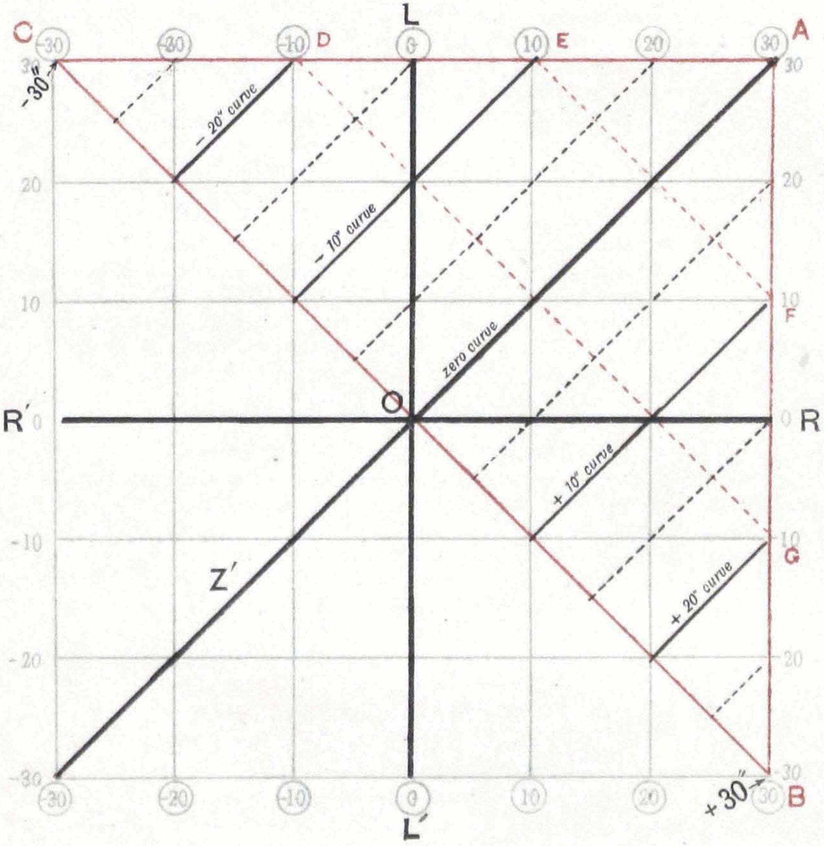

C
-30
-20
-10
D
L
0
10
E
20
30
A
-30″
-20″ curve
-10″ curve
zero curve
+10″ curve
+20″ curve
F
G
R′
R
O
Z′
30
20
10
0
-10
-20
-30
+30″
B
L′

PLATE III.

Diagram of Level No. 9 (Holmes')

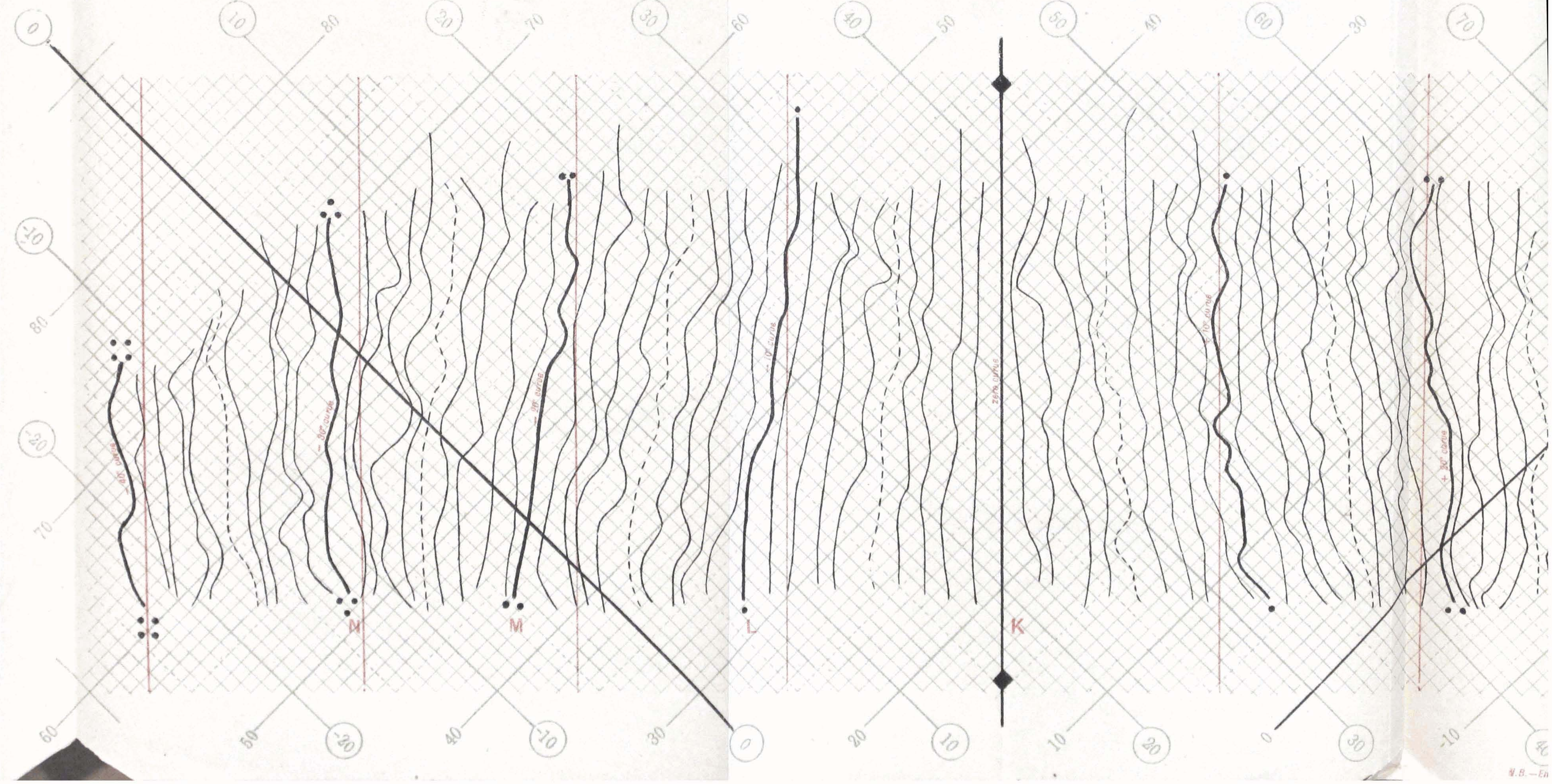
Spirit

Mean value of 1 division = 0".96 about

Using a vertical brass scale 1 division of which = 0·05 inches. The zero of this scale was ·3 divisions to the right (or reservoir end) of the zero point on the tube.

Zenith Telescope.

N.B.—The figures enclosed in circles show the right (or reservoir) end readings of the bubble.



N.B.—En

PLATE III.

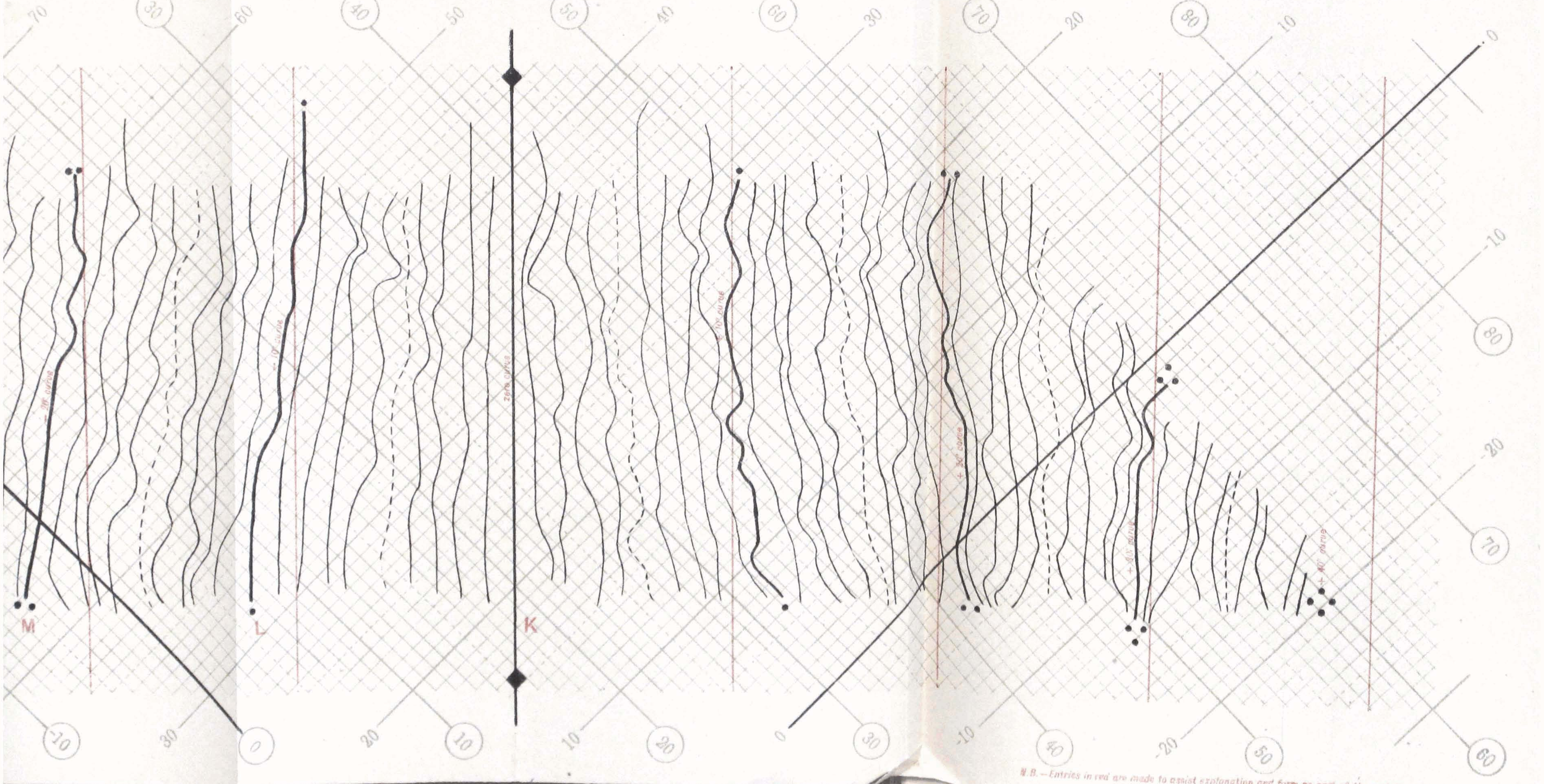
Diagram of Level No. 9 (Holmes')

Spirit

Mean value of 1 division = 0".98 about

Using a vertical brass scale 1 division of which = 0·05 inches. The zero of this scale was ·3 divisions to the right (or reservoir end) of the zero point on the tube.

Plotted from 3700 observations on Cooke's bubble tester No. 1, simultaneously with Level No. 6 (Holmes') in April and May, 1899.



N.B.—Entries in red are made to assist explanation and form no part of the real diagram.

PLATE IV.

Diagram of Level No. 6 (Holmes')

Chloroform

Mean value of 1 division = 0″90 about

Using a vertical brass scale 1 division of which = 0·04 inches. The zero of this scale was 5 divisions to the right (or reservoir end) of the zero point on the tube.

Zenith Telescope.

N.B.—The figures enclosed in circles show the right (or reservoir) end readings of the bubble

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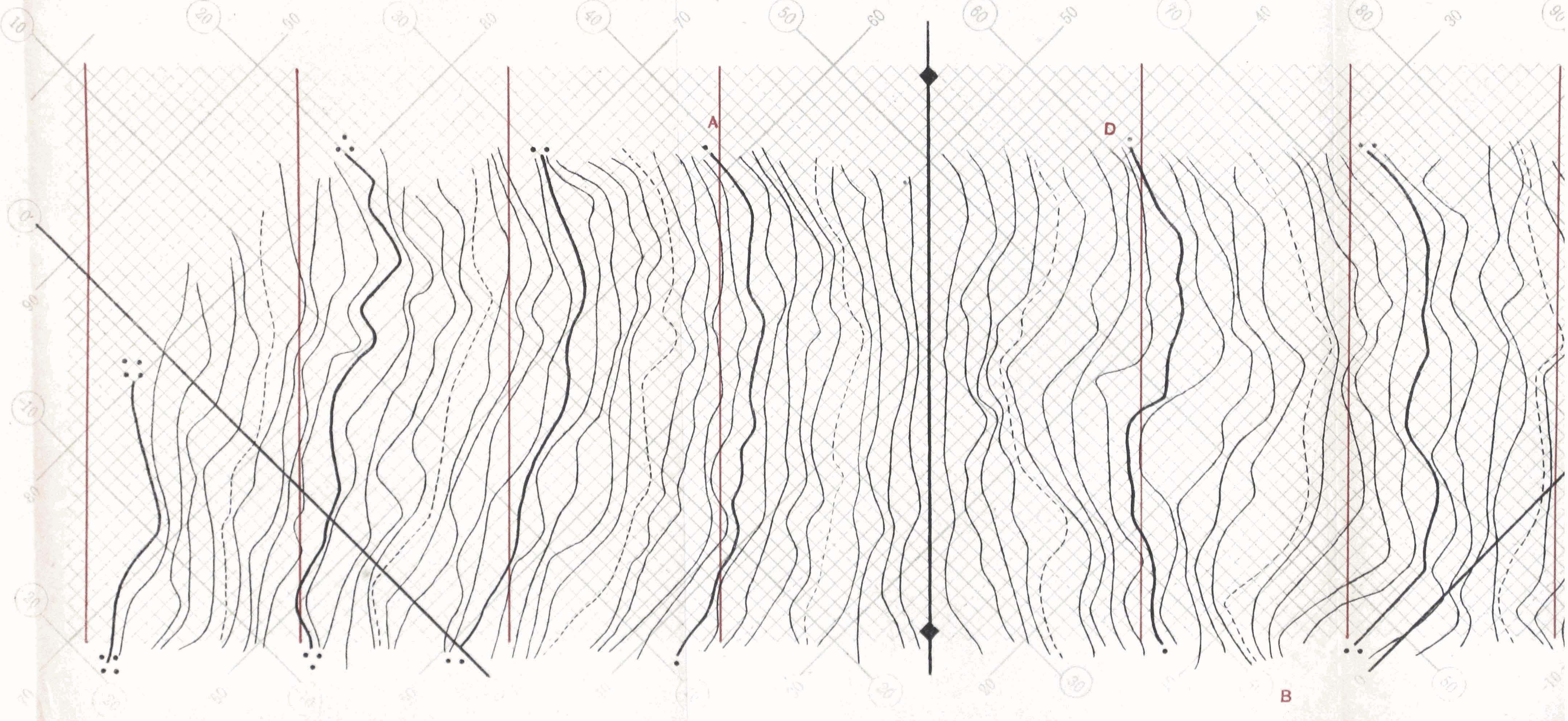


PLATE IV.

Diagram of Level No. 6 (Holmes')

Chloroform

Mean value of 1 division = 0″·90 about

Using a vertical brass scale 1 division of which = 0·04 inches. The zero of this scale was 5 divisions to the right (or reservoir end) of the zero point on the tube.

Plotted from 3700 observations on Cooke's bubble tester No. 1, simultaneously with Level No. 9 (Holmes') in April and May, 1899.

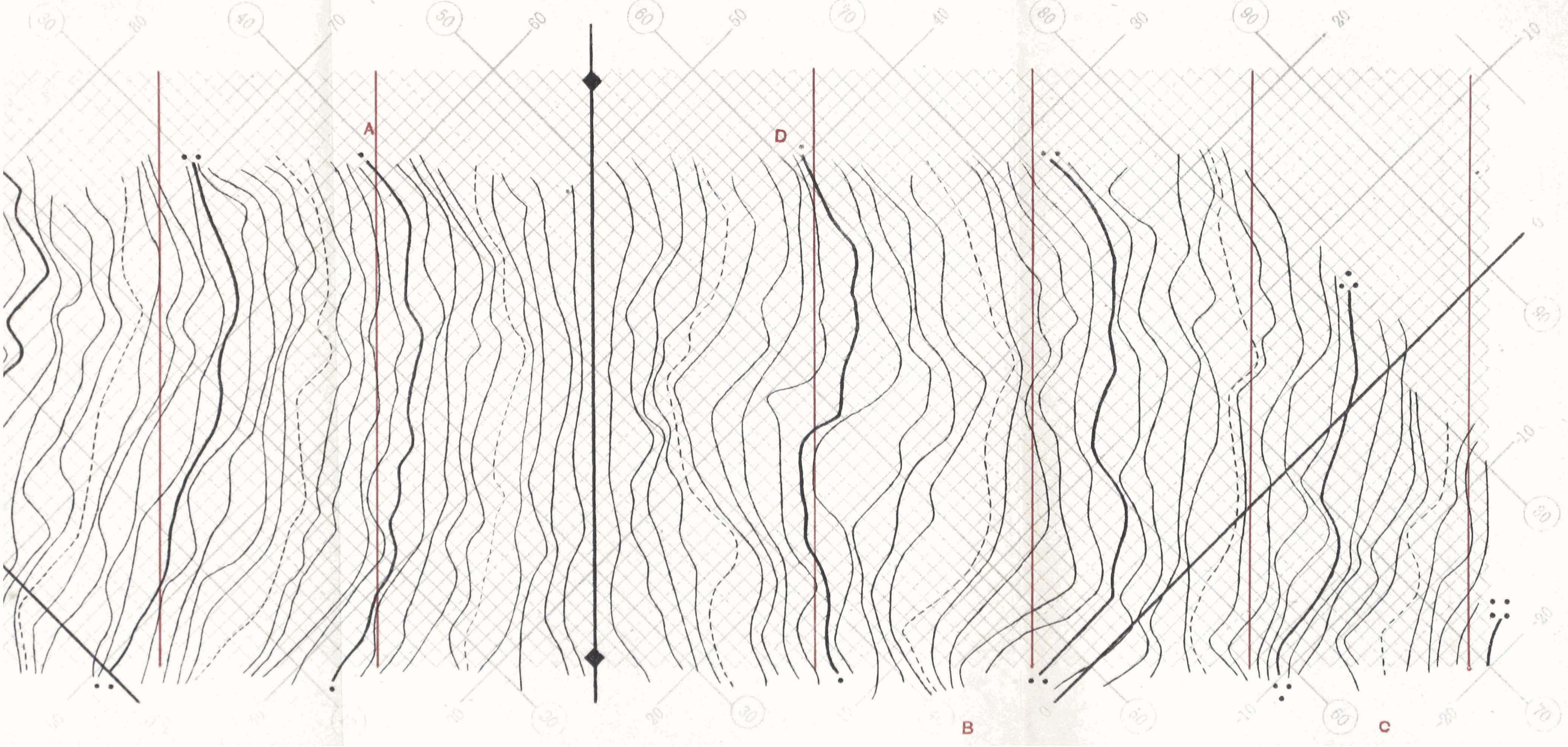
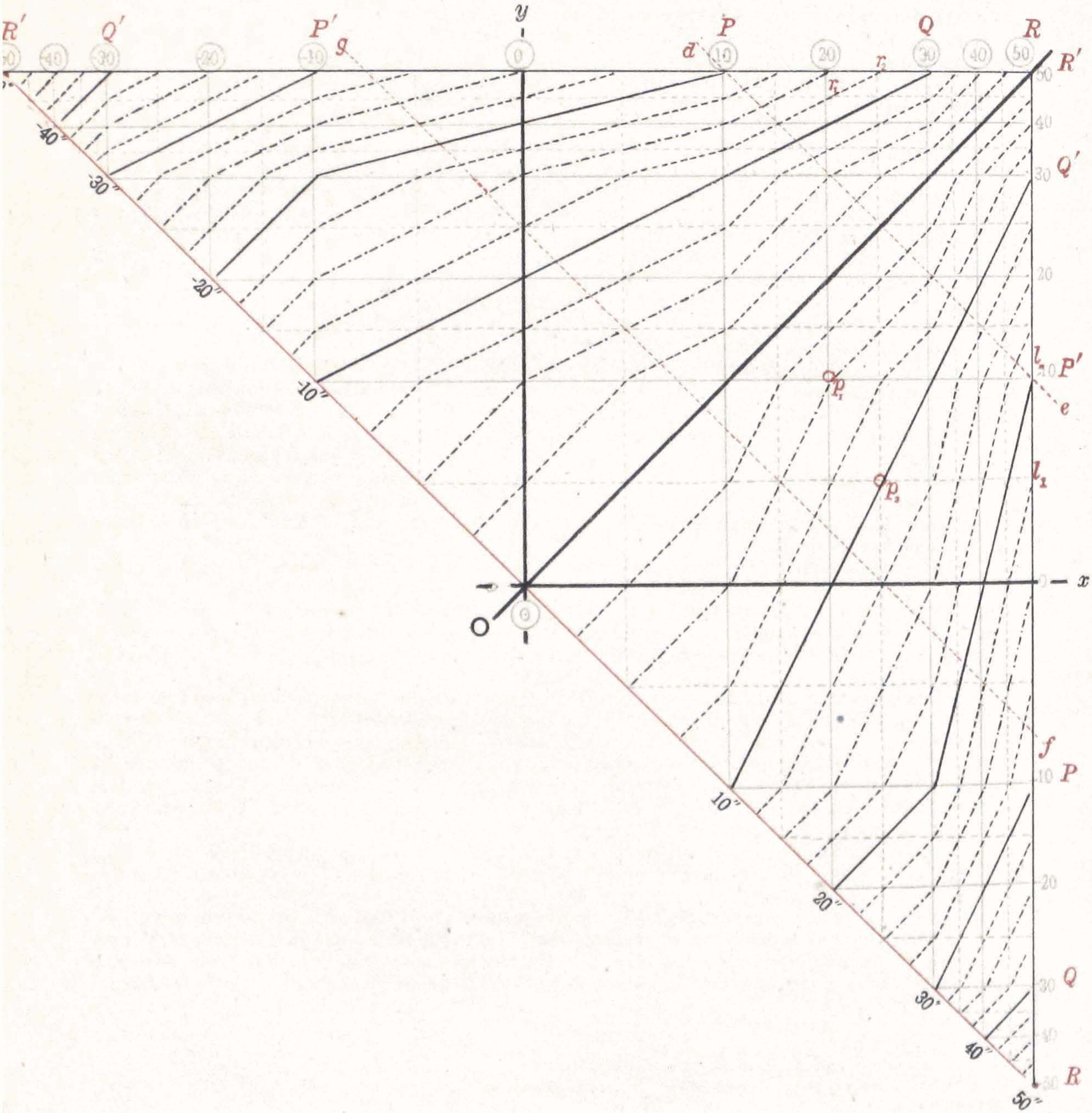


Fig. 4



R'
Q'
P'
g
y
P
Q
R
d
-50
-40
-30
-20
-10
0
10
20
30
40
50
r₂
r₁
R'
Q'
P'
l₂
e
l₁
p₁
p₂
x
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f
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Q
R
-40"
-30"
-20"
-10"
10"
20"
30"
40"
50"
-10
-20
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-40
-50

Let us now consider the continuous red lines joining the points ABC. It is supposed that in this case the limits of the scale or other physical causes confine the bubble readings to 30 divisions on either side of zero, so that there are only 60 available divisions in all, and when either end of the bubble passes beyond these the reading cannot be taken; so that, as AB is drawn along the 30th division on the right hand side, it is evident that it is the limit of the diagram in this direction, for any point beyond it would have to have its right hand reading more than 30, and, by hypothesis, a reading of over 30 is impossible; similarly AC, which is drawn along the + 30 division of the left hand readings, is also a limit of the diagram.

Again, BC will be seen to be the "bubble-length" of a bubble of 0 divisions length; for, if we take any point on it, we shall find that its co-ordinates, *i.e.*, the right and left readings, are always equal and of opposite sign, and (as we know that on the actual level scale a point which gives a positive reading for the right end of the bubble gives an equal negative reading for the left end, and *vice versâ*) this fact shows that the two ends of the bubble are coincident, and the bubble may be regarded as a dot; if such a bubble existed it would of course be able to indicate arcs up to $30''$ on either side of zero, without either end running off the scale, which fact is shown in the diagram by the intersection of the curves with the bubble-length BC of this bubble, which show arcs up to $30''$ on either side of 0. Any continuation of the diagram towards Z' would imply bubbles of a length less than 0, so it is clear that BC is the limit of even our theoretical diagram in this direction.

Again, the point A represents the zero position of a bubble of 60 divisions in length; it is evident that as this is the whole length of our scale, such a long bubble would be useless, as the slightest movement from its zero position would run one end beyond our limits; this fact is shown by the bubble-length of the 60 division bubble being merely the point A. Similarly, ABC having been shown to be the limits of our diagram, it will be seen that the bubble-lengths of bubbles nearly as long as A are very short, indicating that such bubbles have very small play; while as we get to shorter bubbles we find the arcs contained by their bubble-lengths becoming greater and greater, until finally the evanescent bubble at BC is capable of showing as many seconds as there are divisions on the scale.

In actual practice a very long bubble is nearly useless owing to its small play, and on the other hand it will be found that small bubbles become so indeterminate that none of their indications can be relied on, and they are quite useless;* so that, if we decided in this case only to examine bubbles whose lengths were between 20 and 40 divisions, our resultant diagram would be the strip DEFG (*vide* Plates III and IV for specimens of actual diagrams).

Now let us suppose that instead of this ideal case we have an actual curve to deal with, and that we know all its irregularities, so that *instead of graduating the scale on the tube into equal divisions, we are able to graduate it so that every division shows one second of arc at that portion of the curve over which it lies.* Let us mark off the divisions of this irregular scale on the axes Ox and Oy (fig. 4), and draw ordinates through the points so obtained. Then, if we join up the particular intersections (x, y) and $(x + 5, y + 5)$ throughout the diagram, we shall have the curves of the diagram of this level; for, take the points p_1 and p_2, then, since $r_2 - r_1 = 5''$ and $l_2 - l_1 = 5''$, the bubble has moved through an arc of $5''$ between the positions p_1 and p_2; and this agrees with the indications of the diagram curves, which show p_1 as $+ 5''$ and p_2 as $+ 10''$.

It may be said with justice that we are here assuming a definite law connecting the movement of the bubble with the values of the arcs traversed by its ends; so I may at once state that my system of making actual diagrams does not involve any such assumption, but displays results as actually found; but we are now dealing with ideal cases where this method is very convenient,

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* In my levels, bubbles of less than 2 inches length became hopelessly indeterminate; and also short bubbles always took much longer to settle down than longer ones. It would be interesting to try the effect of having levels of greater length, which would permit of longer bubbles being used; as, with an equally true curve, this might afford greater determinateness.

and suffices to show us what *sort* of diagrams certain assumed irregularities would give us; and, as I shall now proceed to show, a consideration of these is most interesting and instructive.

Let us first consider figs. 4 and 5; here I have assumed a tube which is symmetrical on both sides of 0, but in which the central portion *PP′* has a radius of curvature 4 times as great as the end portions QR and Q′R′, with intermediate portions PQ and P′Q′ having a radius equal to the mean of the two extremes, *i.e.*, half that of *PP′* or double that of QR and Q′R′.

Then it is evident that if $OP = PQ = 2QR$ in length, and $OP = 10''$ in arc, then $PQ = 20''$ and $QR = 20''$. This is all duly shown on the axes Ox, Oy, fig. 4, only the graduations have been figured on the outside of the diagram, instead of on the axes themselves, for greater clearness. The resulting diagram then becomes as shown in fig. 4; and fig. 5 shows a very exaggerated drawing of the curve of the tube itself; in the diagram I have entered $2\frac{1}{2}''$ curves as well as the $10''$ and $5''$ curves, simply to show the detail more fully.

Now, as I have pointed out, any actual diagram made of this level would probably lie roughly between the bubble-lengths *de* and *gf* and would consist of the strip *defg*; if we examine this strip we shall first of all see that our diagram *does not differ very greatly from the ideal diagram in fig. 3, considering what very large variations of curvature we have assumed in the tube;* the chief effect noticeable is that the curves converge inwards towards the zero line as the bubble gets longer.* Again, a further examination of the strip *defg* will show that on the long bubble *de* the distance between the curves hardly varies at all, whereas the short bubble *gf* shows marked variations, having the curves much further apart at the centre than at the ends; and, further, the mean value of a division on the short bubble is evidently on the whole much less than that of the long bubble. The physical reason of these effects is obvious; for the long-bubble has not sufficient play to ever get either of its ends on the flattest portions of the curve, so that it can never attain to the very small mean value which the short bubble enjoys when its ends are moving in the flattest portions of the tube; and the intervals of the long bubble are very regular because when one end is on the medium portion of the curve the other is on the steepest portion and *vice versâ*, so that the mean curve traversed by the two ends is always the same, except just at the centre of its run, where *both* ends are on the medium portions simultaneously; it will be seen that the diagrams duly indicate this on the bubble-length *de* by the two $5''$ curves on either side of the centre having a larger interval than the remainder of the curves, where one or other end is always on a steep portion. I have drawn the bubble *de* to scale in its central position on my exaggerated tube (fig. 5), a glance at which will show what I mean. The short bubble *gf* which I have also drawn in its central position in fig. 5 will be seen to just have both its ends in the flattest portion of the tube when in this position; this fact is shown in the diagram by the large intervals of the $2\frac{1}{2}''$ curves on either side of 0 on the line *gf*. On running this bubble a little more to either side, it will be seen that for some distance one end will always be on the flattest portion *PP′* while the other is on the medium; the intervals on the diagram are accordingly constant throughout this period. Eventually, at the end of its run, one end of this short bubble gets on the steepest portion, and we have a corresponding contraction of the intervals on its bubble-length, in the diagram.

I have gone into this matter at some length, because, if we now turn to plate III and examine the actual diagram of level No. 9, which is the result of simple observation without reference to any theories whatsoever, we shall find the whole of the above conditions fulfilled. In this diagram it will be seen that the ordinates of the divisions are printed diagonally in order to bring the diagram (corresponding to the strip *defg*) conveniently on the paper; so that to compare it conveniently with the strip *defg*, fig. 4 should be held so as to bring the lines *de* and *fg* "horizontal", so to speak. The first similarity we may recognize is the general convergence of

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* Other observers in this department have made rough diagrams of their levels, and it is a curious fact that nearly all show signs of this peculiarity; so I am inclined to think that the process of manufacture may be such as to cause the central portion of the tube to have generally a flatter curve than that at the ends.

Fig. 5

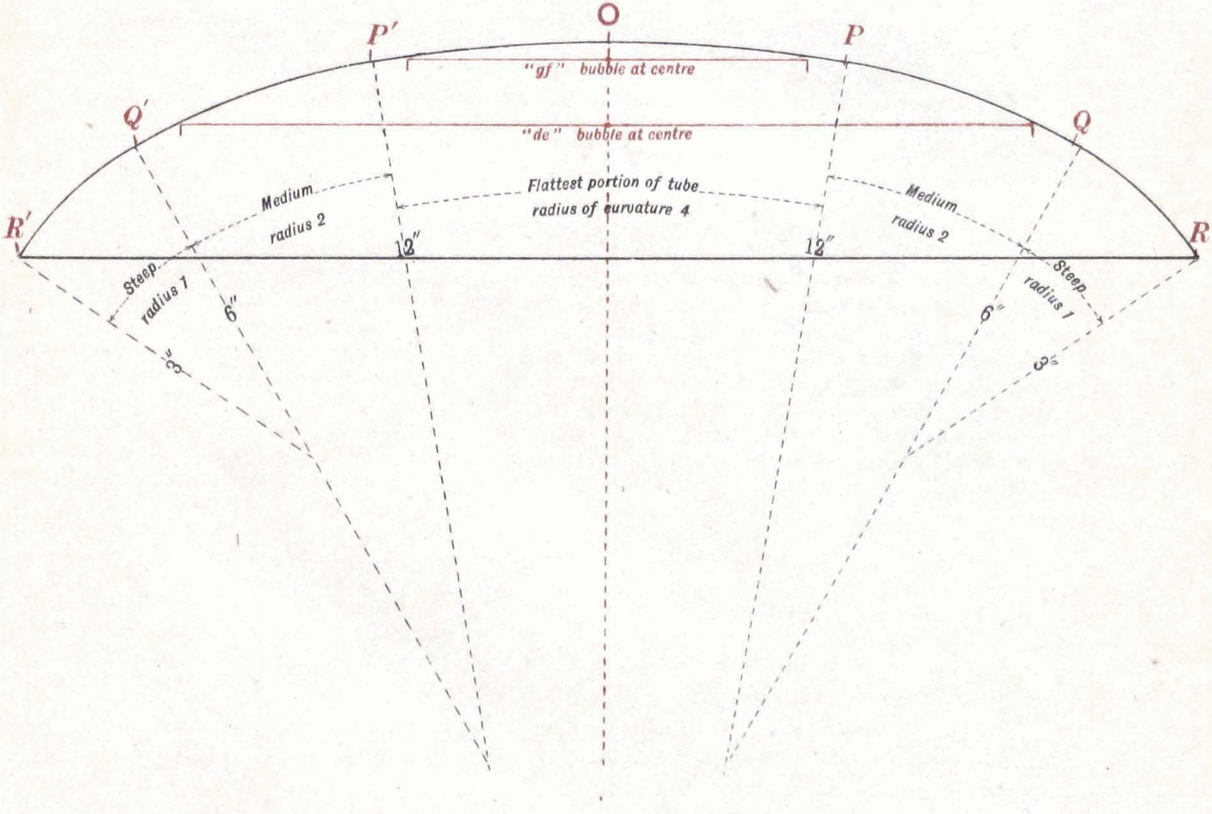

O
P′
P
Q′
Q
R′
R
"gf" bubble at centre
"de" bubble at centre
Flattest portion of tube
radius of curvature 4
Medium
radius 2
Medium
radius 2
12″
12″
Steep
radius 1
Steep
radius 1
6″
6″
3″
3″

Fig. 6

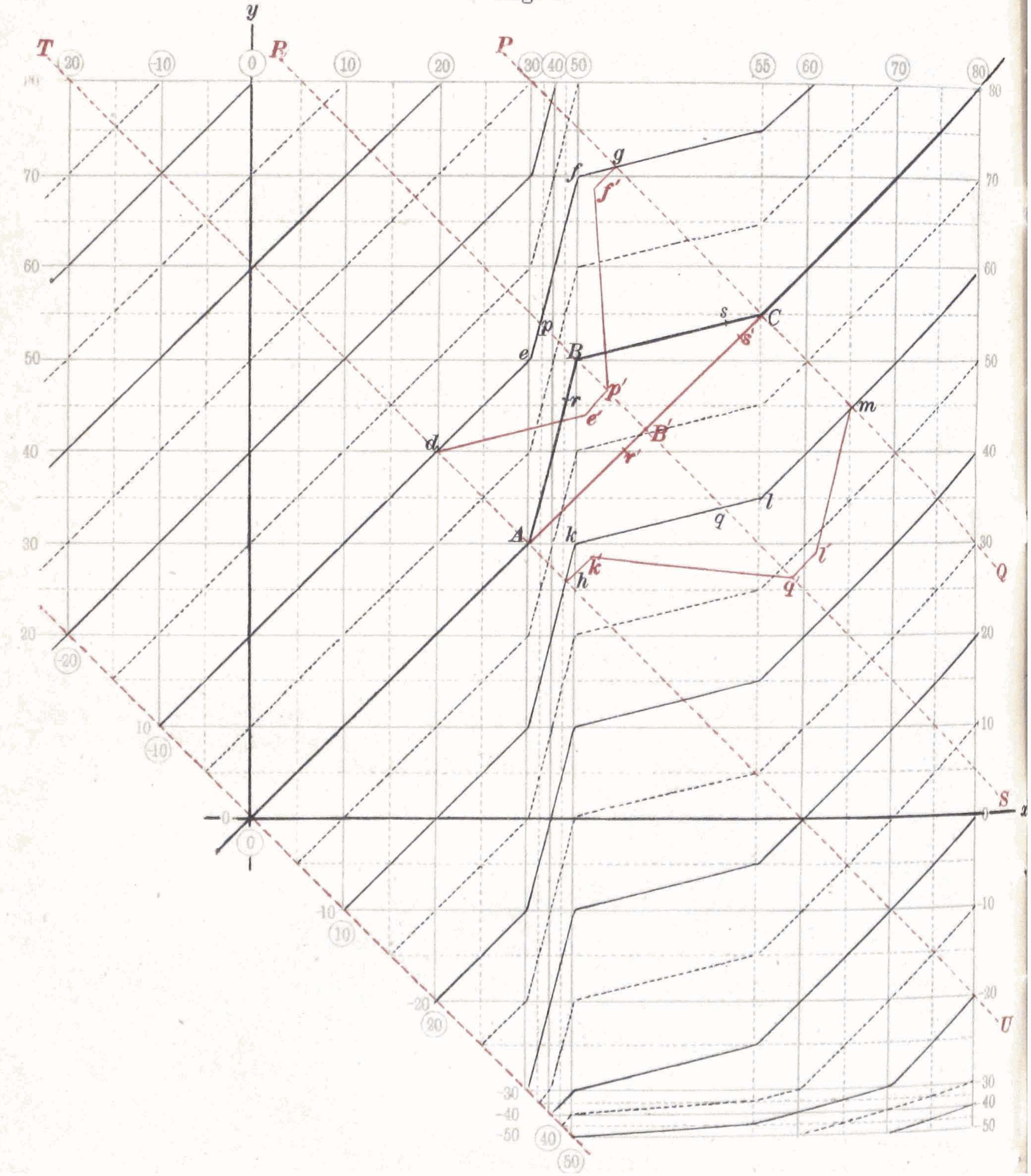
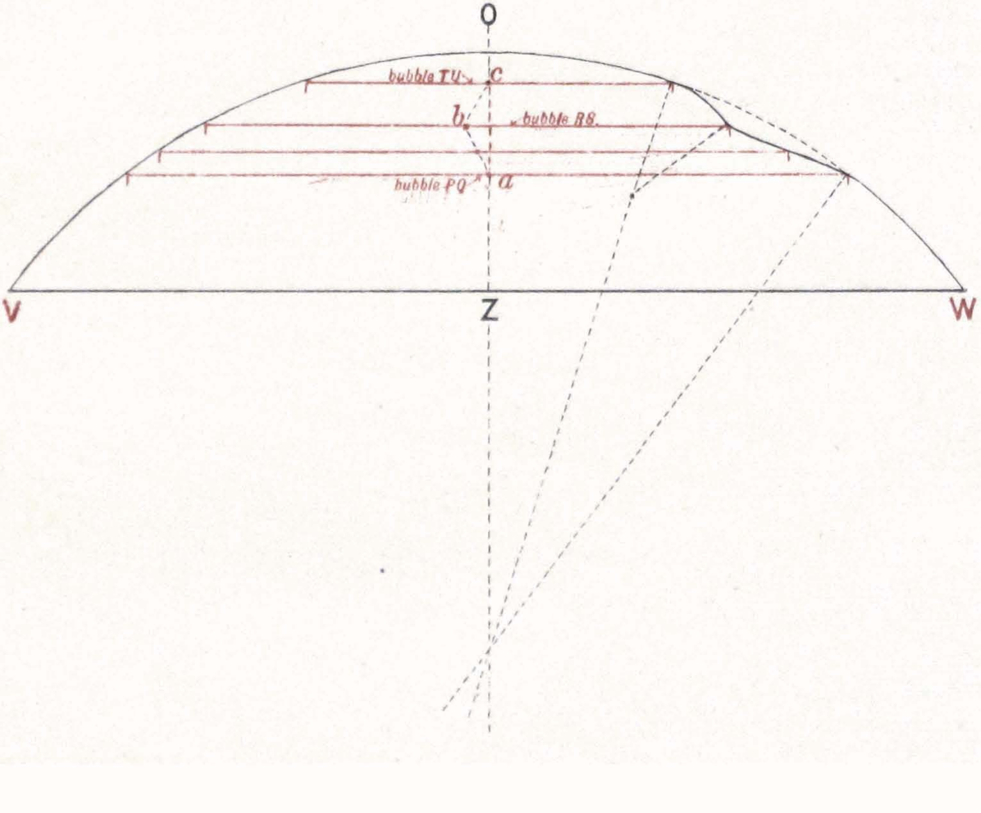

y
x
T
R
P
Q
S
U
A
B
C
d
e
f
g
h
k
l
m
p
q
r
s
f′
p′
e′
B′
r′
s′
k′
q′
l′
-20
-10
0
10
20
30
40
50
55
60
70
80
-30
-40
-50

Fig. 7



the curves towards the zero lines as the bubble lengthens; then we see that at the top of the diagram (*i.e.* along the bubble-length of the largest bubble) the intervals between curves are fairly equal; whereas with the shortest bubble at the bottom of the diagram, the intervals are very much larger at the centre than any of the above, and also that they vary immensely amongst themselves, exactly as in the strip *defg*, diminishing as the bubble gets nearer the ends of the tube, (*e.g.* compare the intervals KL, MN, which are respectively 3·6 inches and 2·3 inches in length though they both equal 10″). I think therefore we have fair grounds for supposing that No. 9 level is similar to that imagined in fig. 5 and has a much flatter curve at the centre than at the ends, and that the variations in the radius of curvature are really very considerable.

Let us now turn to another imaginary case shewn in fig. 7; here we shall find that a serious complication is introduced, which depends on the assumption of an unsymmetrical tube, *i.e.*, one which is not precisely the same on both sides of 0. To simplify the question as far as possible I have assumed a perfect tube such as that shown in fig. 3, marred only by the fact that on the right-hand side a sort of dimple occurs in the curve, the first part of which is very steep, having a radius of curvature one quarter that of the general mean, and the second part straight, bringing the surface back to the normal curve.

The resultant diagram together with an exaggerated sketch of the curve itself are shown in figs. 6 and 7. No attention need be paid to the red lines on the diagram at present; but an examination of the true curves, drawn in black, will show how the intervals decrease when one end of the bubble is on the steep portion, and how they increase when it is on the flat portion, which of course is what we should expect; it may also be noted how at the steep portion, all the curves become more perpendicular to the axis *ox* on which the steep part is plotted, and how at the flat portion they become more parallel with it; the same applies to the lower part of the diagram, where the fault appears among the minus readings of the left end and is therefore plotted on the axis *oy*; this latter portion is however mostly out of the practical part of the diagram as it only comes in amongst the very short bubbles, and a glance at fig. 7 will show that only a very short bubble could get its left end into the irregular portion of the curve.

The main point of interest, however, in fig. 6, is the bend in the zero curve shown at ABC. This shows that *in an unsymmetrical tube, if we refer the whole diagram to any given dislevelment as zero, we get a zero line which is not straight.* The physical cause of this is shown in fig. 7, where I have drawn in red the positions which would be taken up by several bubbles with the tube in its "horizontal" position VW. (By this I mean that the slope of VW is such that all bubbles lying in the *normal* parts of the curve would have their centres on the zero line OZ when the tube was in the position VW). Now if we bisect the 4 bubbles shown it will be seen that when one end is on the irregular portion of the tube the centre of the bubble does not lie on the zero line OZ, but deviates towards the left of the tube as shewn at *abc.* That is to say, supposing we were working with an equally divided level scale, all our bubbles would be at "equal readings" when our tube was in its zero position VW, except those bubbles whose length was intermediate between PQ and TU, which would have their right-hand readings less than their left when the tube was in its zero position.

This is exactly what is indicated in the diagram by the deviation ABC. It is therefore evident that, unless a tube is exactly symmetrical on each side of the centre of the scale, the true zero line will not be straight, and *that, in assuming the position of equal readings as the zero position for each bubble, and so forcing the zero line into a straight line, we are in reality using a different, or at all events independent, zero for each length of bubble.*

I have recognized this blemish in my system from the first, but as I could see no way of discovering the true curve of the zero, and as a straight zero involves no error unless we happen to be comparing the positions of two bubbles of different lengths,[*] I made my diagrams with a straight zero. I have since devised a simple way of finding the curve of the zero (*vide* Appendix I), and the preliminary determination of this will enable the whole diagram to be consistently calibrated to any assumed zero position of the level.

Before preparing the imaginary diagrams now under discussion (an idea which only occurred to me quite recently), I had not adequately realised the full value and importance of this further refinement in improving and simplifying the whole calibration. I will go into this point more fully later, and will at present only discuss the sort of distortion which the straightening of the zero line may produce in a diagram. To show this I have prepared fig. 8; this is a diagram of exactly the same level as that shewn in figs. 6 and 7, only plotted with a straight zero and showing all the distortions consequently introduced. By comparing it with fig. 6, it will be seen that distortion only occurs between the bubble-lengths PQ and TU, that is, in that portion of the diagram in which the zero has been artificially displaced. The condition on which this diagram has been constructed is that the intervals along any particular bubble-length should be exactly the same as in the true diagram, fig. 7, only that the zero is displaced from the true curve ABC to the straight line $AB'C$ and every other point on each bubble-length is displaced similarly and equally to the displacement of its zero. I have shewn in both diagrams the curves $d\,e\,p\,f\,g$ and $d\,e'\,p'\,f'\,g$ to make my method clear, showing that $er = e'r'$, $pB = p'B'$, and $fs = f's'$, which satisfies the above condition; and similarly with $h\,k\,q\,l\,m$ whose displacement to $h\,k'\,q'\,l'\,m$ is also shown on both diagrams. This system is not absolutely correct, as the displacements instead of being lineally equal should be actually equal in seconds of arc, as shown by the intervals on the bubble-length; the maximum error introduced owing to this cause is at p', for, whereas BB' only equals about 5″, pp' = about 12″, as I have plotted it, (*vide true* diagram, fig. 6) and p' should really lie somewhere between p and B; this is however an extreme case, and in most places the deviation from theoretical accuracy is very slight, so I have considered the simple method I have adopted quite sufficiently accurate to give a rough idea of the distortive effects of straightening the zero, and, as this is all we want, I shall now proceed to discuss it as it stands.

It may first be noted that the indications of our steep bit of curve almost disappear in the vicinity of the zero line, *e.g.*, the + 5″ and − 5″ curves become more or less parallel to the zero line where they should be steep; again we find that above the zero line the indications of this steep bit are shifted somewhat to the right; another result we may note is that the apparent indications of our flat portion have become much exaggerated, *e.g.*, $k\,q$ has been still further flattened to $k'\,q'$; then there is the very obvious effect of the marked bend toward S introduced into *every* curve as it crosses the bubble-length RS, where the zero has suffered its maximum displacement; and, finally, we may contemplate the chaotic appearance of fig. 8 as compared with the simple and easily interpreted curves in fig. 6. When it is remembered that all this chaos is due to one simple fault in the tube, it may be realised how difficult of interpretation will be an actual diagram, subject to the combined effect of endless and varied irregularities on both sides of 0, and how seriously this difficulty is enhanced when we have arbitrarily calibrated to a straight zero as I have done in my diagrams. I am however able to show marked indications of just such a fault as that above described on the right hand side of my level No. 6. If we turn to the actual diagram of this level (Plate IV), and run our eye along division 40 on the right hand side (*i.e.*, the line AB) we shall see the following effects between divisions 38 and 42 which we may compare with those of the assumed steep bit in fig. 8:—

(1) On the right, or lower, side of the zero line the curves mostly get close together and run almost parallel with division 40 at the points where they cross it.

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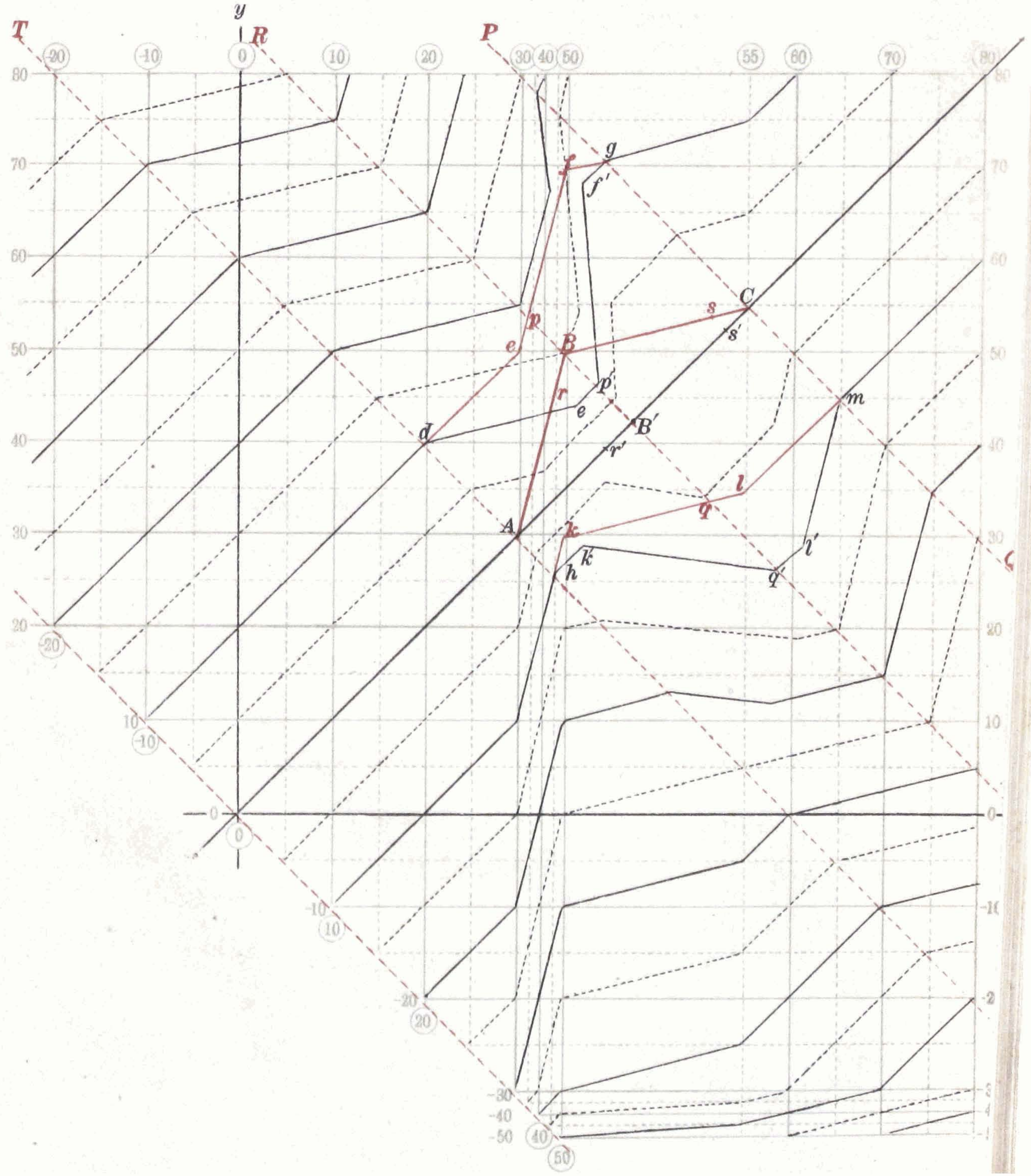
[*] In actual work this would only occur appreciably with a rapidly changing temperature, or when, through a jerky movement of the instrument, we happened to throw off a satellite bubble, and so to lose length, and the result in this latter case would be doubtful anyhow; as a general rule the two readings to be compared are taken in fairly quick succession so that the bubble is practically the same length in the two positions, in which case it is evident that both readings will be on the same bubble length and so will be referred to the same zero, *i.e.*, to the position of "equal readings" of that bubble, and no error whatever will be involved.

Fig. 8


y
T
R
P
Q
(-20)
(-10)
(0)
(10)
(20)
(30)
(40)
(50)
(55)
(60)
(70)
(80)
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
(-20)
(-10)
(0)
(10)
(20)
(40)
(50)
g
f
f′
C
s
s′
p
e
B
p′
r
e
m
d
B′
r′
l
q
A
k
l′
k′
h
q′

(2) The curves near the zero lose all signs of this and run fairly parallel to the zero line, but still it will be seen that their intervals from the zero are generally less near the points where they cross the 40 division.

(3) To the left of, or above, the zero line the steep portion of the curve reappears, but it will be noticed that it is generally thrown forward to division 42 about, whereas on the right of the zero its maximum was about at division 39.

These indications all agree with those noted in fig. 8; and if we look further to the indications of the actual diagram between divisions 40 and 60 on the right-hand side, *i.e.* the strip ABCD, we shall see a remarkable likeness to the flat portions of the diagram on fig. 8. There is therefore good reason to suppose that we have a kink in level No. 6 near division 40 on the right, followed by a flat portion; the presence of this, as well as of many other trifling irregularities, would doubtless have become much more clearly apparent if I had calibrated to a "true" instead of to a "straight line" zero.

It is evident that, in a constant temperature, it is the *volume* of a bubble, and not necessarily the *length* which remains constant; so that in an irregular tube, as the mean cross section of the bubble would vary in different positions, its length would certainly vary, even in a constant temperature. With very good tubes such a variation would be necessarily very small, still I have noticed instances of a slight variation in the length of the bubbles of No. 6 level, such as a consideration of fig. 7 would lead one to expect. This is an independent corroboration of the existence of the kink which the diagram indicates. Further it is important to note that I found, as a matter of actual fact in testing level No. 6, that the level indications are bad when one end of the bubble lies on this kink, and are liable to be very indeterminate when it is on the flat part. The remainder of the tube is good; so I am now able to avoid all the more gross errors to which the level is liable, by taking care, in actual work, not to have one end of the bubble under the portion of the scale 38 to 60 on the right-hand side. This is a very valuable piece of information which is afforded by a glance at the diagram; and it is for this reason that I have devoted so much space to these imaginary cases.

PART III—DETAILS OF PROCEDURE.

Plates III and IV show the convenient form of sheet used for plotting the strips of diagram I have calibrated. The divisons in blue were 0·2 inches apart which is a scale that can be read far more accurately than we can pretend to take our bubble readings, and which therefore guarantees that no increased inaccuracy will be introduced in interpreting them.

Organization in sets.

The greatest arcs which even my shortest bubbles would give was about 40″ on either side of zero, so there were about 80 curves to be determined; I divided these into 8 sets, each set consisting of a series of 10 curves, 8″ apart. So that reading from left to right the sets were as follows:—

1st set − 40, − 32, − 24, − 16, − 8, 0, + 8, + 16, + 24, + 32, + 40
2nd set − 39, − 31, − 23, − 15, − 7, + 1, + 9, + 17, + 25, + 33
3rd set − 38, − 30, − 22, − 14, − 6, + 2, + 10, + 18, + 26, + 34
4th set − 37, − 29, − 21, − 13, − 5, + 3, + 11, + 19, + 27, + 35
5th set − 36, − 28, − 20, − 12, − 4, + 4, + 12, + 20, + 28, + 36
6th set − 35, − 27, − 19, − 11, − 3, + 5, + 13, + 21, + 29, + 37
7th set − 34, − 26, − 18, − 10, − 2, + 6, + 14, + 22, + 30, + 38
8th set − 33, − 25, − 17, − 9, − 1, + 7, + 15, + 23, + 31, + 39

In order to avoid the awkwardness of taking minus readings on the bubble tester, I used to set the bubble at "equal readings" with the tester at 40, so that 0 in the above sets was shewn by a tester reading of 40, and any other curve x by a tester reading of $x + 40$. The sets so translated become:—

0,	8,	16,	24,	32,	40,	48,	56,	64,	72,	80,
1,	9,	17,	25,	33,	41,	49,	57,	65,	73,	
2,	10,	18,	26,	34,	42,	50,	58,	66,	74,	
3,	11,	19,	27,	35,	43,	51,	59,	67,	75,	
4,	12,	20,	28,	36,	44,	52,	60,	68,	76,	
5,	13,	21,	29,	37,	45,	53,	61,	69,	77,	
6,	14,	22,	30,	38,	46,	54,	62,	70,	78,	
7,	15,	23,	31,	39,	47,	55,	63,	71,	79,	

and they were identified by their first number, thus the first set here shown would be known as the "0" set and the last as the "7" set. I tested one "set" at a time, turning the micrometer successively to the numbers belonging to the set as shewn above. The interval 8″ was sufficiently large in my levels to give a fair determinate movement to the bubble, and there are several small conveniences in having 8 sets which I need not here trouble to discuss, as no principles of importance are involved.

It will at once be seen that, as every curve in the diagram is included in the above, if we determine every set for every length of bubble included on the diagram, all our curves will be complete. As a matter of fact I determined the points on each set of curves on a number of successive* bubbles varying about 4 or 5 divisions in length, so that my diagram was fairly covered with data, and it only remained to draw in the curves by eye.

Systematic Series for each set.

For every set I drew up an appropriate "series" which should give me 4 determinations of every point in the set, 2 obtained as the result of a left to right motion of the bubble, and 2 from a right to left; it is most important that this condition should be always satisfied in any and every series made, in order to balance the effects of stickiness in the bubble which may prevent it from quite arriving at its new position, or may cause it after oscillating past it to remain beyond it. Further I arranged that two of these determinations should be from arcs of 8″, and two from arcs of 16″, with a view to showing whether the effects of stickiness, if any, varied with the momentum of the bubble, which of course would be greater after moving through a larger arc; and finally the series had to be interspersed with a sufficient number of the "equal readings" (*i.e.*, "40") position, to which all the curves were to be referred. The series I prepared to satisfy all these conditions are shewn in Table A.

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* This word does not apply strictly here, various length bubbles were taken for each series in any order, as happened to be convenient and *not* successively in order of length *e.g.*, the longest first and the next longest next. I state this in case it should be imagined that the gradual variation in behaviour of bubbles as they get shorter might be due to some actual variation in the levels from day to day.

Table A.

Serial No. of Observation	SUCCESSIVE POINTS OF SERIES.							
	For the "0" set	For the "1" set	For the "2" set	For the "3" set	For the "4" set	For the "5" set	For the "6" set	For the "7" set
1	**40**	**40**	**40**	**40**	**40**	**40**	**40**	**40**
2	0	1	2	3	4	5	6	7
3	8	9	10	11	12	13	14	15
4	16	17	18	19	20	21	22	23
5	24	25	26	27	28	29	30	31
6	32	33	34	35	36	37	38	39
7	**40**	41	42	43	44	45	46	47
8	48	49	50	51	52	53	54	55
9	56	57	58	59	60	61	62	63
10	64	65	66	67	68	69	70	71
11	72	73	74	75	76	77	78	79
12	80	**40**	**40**	**40**	**40**	**40**	**40**	**40**
13	72	73	74	75	76	77	78	79
14	64	65	66	67	68	69	70	71
15	56	57	58	59	60	61	62	63
16	48	49	50	51	52	53	54	55
17	**40**	41	42	43	44	45	46	47
18	32	33	34	35	36	37	38	39
19	24	25	26	27	28	29	30	31
20	16	17	18	19	20	21	22	23
21	8	9	10	11	12	13	14	15
22	0	1	2	3	4	5	6	7
23	16	**40**	**40**	**40**	**40**	**40**	**40**	**40**
24	32	1	2	3	4	5	6	7
25	**40**	17	18	19	20	21	22	23
26	48	33	34	35	36	37	38	39
27	64	49	50	51	52	53	54	55
28	80	65	66	67	68	69	70	71
29	64	57	58	59	60	61	62	63
30	48	41	42	43	44	45	46	47
31	32	25	26	27	28	29	30	31
32	16	9	10	11	12	13	14	15
33	0	**40**	**40**	**40**	**40**	**40**	**40**	**40**
34	8	9	10	11	12	13	14	15
35	24	25	26	27	28	29	30	31
36	**40**	41	42	43	44	45	46	47
37	56	57	58	59	60	61	62	63
38	72	73	74	75	76	77	78	79
39	80	65	66	67	68	69	70	71
40	72	49	50	51	52	53	54	55
41	56	33	34	35	36	37	38	39
42	**40**	17	18	19	20	21	22	23
43	24	1	2	3	4	5	6	7
44	8	**40**	**40**	**40**	**40**	**40**	**40**	**40**
45	0	...	...	...	...	...	...	...
46	**40**	...	...	...	...	...	...	...

It will be seen that each of these series gives 4 determinations of every point in its set, (except occasionally for the last points at the end of the tube which are not important) together with a sufficient number of zero determinations interspersed. Also, after setting at 40, every series commences with the initial number of its "set"; we then run up the scale and down again in arcs of 8″ length, putting in a zero determination after each run; this is followed by two runs up and down of arcs of 16″ length, with zeros interspersed as shown. Long bubbles naturally run out of sight near the ends of the series, as they have not sufficient "play" to be able to show arcs of 40″ on either side of zero; *e.g.*, in the specimen record on page 16, it will be seen that both bubbles ran out of sight in the micrometer position 79.

Plotting on separate sheets and final abstraction.

The result of trying to plot all the points thus obtained on to one sheet would have been confusing, so I used 4 preliminary sheets for this purpose, plotting my "0" series and "4" series on one sheet which would accordingly be known as the "0, 4" sheet, and my "1" and "5" series on the "1, 5" sheet, and so on, having two sets with 4″ intervals between them on each sheet. Having plotted all my data, I drew in the curves on each sheet, and then abstracted the curves from all the sheets as they stood into a fifth or "abstract" sheet; having copied all the curves on to this sheet, I marked those portions of each curve actually determined by a colour appropriate to its "set", making these marks heavy or light in proportion as the data on which they depended were concordant or the reverse; this abstract sheet thus gave an idea of the "weight" of the different portions of every curve, and from it I made out my final diagram, adjusting conflicting data according to their weights and to the best of my judgment.

I had originally intended to publish these sheets to enable the reader to judge for himself of the consistency of the work, but have abandoned this intention owing to the large amount of labour which would have been entailed in the printing and registration of the plates; any observer can easily discover the wonderful accordance which is observable in the data, by running two or three sets, (with bubbles of a proper length) and plotting the results on section paper. I rely on the figures in Appendix III to show that the variations of mean value exhibited by my diagrams are in keeping with the actual facts; so it will suffice to say that the consistency of the observations, considering all the disturbances to which they were liable, seems to me nothing less than astounding; for, not only were the different determinations of a point over and over again coincident, but, in the abstract diagram, the different points of adjacent curves, which were taken on different days, or even weeks, hardly ever showed any interlacement except in the bad parts of No. 6 level; in fact in No. 9 level they were so consistent that, given, say, the even number curves, the remainder might have been simply interpolated with hardly any loss of accuracy.

Enhancement of difficulty due to straight zero.

In drawing the final diagram there is often much room for judgment in deciding what curves to adopt when the data are conflicting. I completed my diagrams before I had fully realised the distortive effects of a "straight-line" zero, and even if I had appreciated them the matter is too complicated to be easily grasped; for this reason I am sure my final curves are not so true as they would have been if I had calibrated to a "true" zero, thus getting more clear and simple evidence of such irregularities as exist, and so having a reasonable hypothesis to assist me in adjusting conflicting data. The effects of this blemish were much more felt in dealing with level No. 6, where the irregularities are more varied, and the curve less symmetrical, giving in certain parts of the tube very puzzling interlacements of adjacent curves.

Effect of temperature and other details.

In order to examine the effects of temperature, if any, I always worked the series 0, 2, 4 and 6, in the early mornings when the temperature varied between 61° and 86°, and the series 1, 3, 5 and 7 at midday with a temperature of 80° to 100°; had there been any appreciable temperature effect we should have had distinct evidence of the "even number" curves being generally further from, or nearer to, the zero than the "odd number" curves; there is no sign of such an effect in either level, and the comparison of the results of two adjacent series one of which was taken at 61° and the other at 99° has convinced me that temperature within this range has absolutely no appreciable effect.

In plotting the points of different series I used to write their serial number beside them, with a view to hereafter examining the questions suggested above under the heading "systematic series for each set"; but I do not think my method was sufficiently matured at the time to afford very reliable evidence, so I have not attempted to enquire into these points any further for the present. I also ran a couple of series to try the effect of using the level at a tilt, that is by turning the tube about its own axis so as to bring the scale more forward or more back as the case might be. I could find no effect in a turn of this kind of one or two degrees; with a turn of 6° in No. 9 level, the results were all very accordant amongst themselves but showed distinct displacement in the diagram; this is a matter which requires more careful investigation with better diagrams than mine.*

Careful experiment showed that it was desirable to allow 40 seconds for the bubbles to settle before attempting to read them.† I used to have a watch beside me and was careful to always allow a constant interval throughout a series, and no interruption of work was permitted until the series was finished. As it takes about 15 seconds to read the levels, the observation of a series occupies about three-quarters of an hour.

Corrections for zero movements.

The above system is all obvious and simple enough, but in carrying it out there is a serious practical difficulty now to be considered. I have already referred, at the end of Part I, to the innumerable instrumental disturbances which tend to vitiate all level testing; and the natural result of these, in working a "series," is that when we return to our 40, or zero, reading, we shall probably find our bubble has moved out of the position in which it was originally set, and generally, all the zero determinations will differ slightly from one another.

My object throughout was to refer every position of each bubble to its "equal readings" position as zero. I have discussed the errors involved in this, and there is no doubt that it would be better to first determine the curve of the zero line, as suggested in Appendix I; this curve would afford a point for every bubble-length to which the readings of that bubble could be referred, and, in working a series, the bubble would then be started from its own proper zero, which would not necessarily be the position of "equal readings". Failing such a curve, however, the assumption of "equal readings" as the zero of every bubble is the only convenient one. In any case, the difficulty of the zero moving during a series will be the same whether that zero is assumed at "equal readings" or at any other position; so I will simply discuss it as I had to deal with it, *viz.*, as a reduction of the zero to "equal readings", merely premising that the problem is the same whatever our desired zero position may be.

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* Some means of telling the exact "tilt" of the tube within a degree or two is much required; a diminutive cross level of very small radius of curvature welded on to one end of the tube would do, but seems rather clumsy. It is important that the cross levelling of the tube while being tested should be identical with that when mounted on the instrument, and at present this condition can only be roughly attained.

† This of course varies with every level; also with very short bubbles I had to allow more than 40 seconds, whereas with very long ones I found 30 seconds ample.

To begin with, it is almost impossible to set a bubble exactly at "equal readings", and, even if we attain this, it will be found that the various zeros taken during a series have not kept to it. It is evident that it will not do to simply plot the result so found, as for every series we should find our zero displaced from the straight line in which it should lie; such a displacement would of course in no way represent the "true" zero of the bubble, as it is merely the result of fortuitous discrepancies from our attempts at setting the bubble at "equal readings," and it would only further complicate matters to plot it on the diagram; a still greater objection to doing this is that *in the same bubble-length* we might have 8 different zeros corresponding to the 8 sets of curves; so that the "5" set might be referred to a zero 1″ to the right of equal readings, and the "6" set to one 1″ to the left, and so on, each set having that zero which may happen to have been evolved from the particular series by which the set was determined on the bubble-length in question. Such a state of affairs would of course be absolutely useless, so it is evident that we must take each series separately and reduce its zero to "equal readings," and displace all its other determinations similarly and equally, so as not to disturb the intervals between curves by the displacement.

The simplest way of doing this would be to take the mean of all the zero determinations in a series, and so to discover what correction is necessary to reduce the mean to "equal readings" and then to apply this correction to every reading in the series. This method would be not only the simplest but the best if the variations in the zero determinations were *entirely* fortuitous and irregular; but my experience is that they are largely due to persistent and more or less regular disturbances; to deal satisfactorily with these it is evident that we should discover the correction required to bring each zero determination to equal readings; and then, assuming the changes in these corrections to be due to a gradual and regular disturbance, we should interpolate arbitrary corrections for the intermediate observations, increasing or decreasing by progressive amounts as indicated by the two zero determinations between which they lie; *e.g.*, if the 23rd observation in a series is a zero determination requiring a correction of + 0″·5 to bring it to equal readings, and the 33rd is another requiring a correction of − 0″·5; then, as our observations are always taken at equal intervals of time, we assume that one end of the tube has risen 0″·1 between each of the 10 observations; so that, having applied corrections of + 0″·5 and — 0″·5 to observations 23 and 33 respectively, we get by interpolation the following corrections for the observations 24 to 32:— + 0″·4, + 0″·3, + 0″·2, + 0″·1, 0, — 0″·1, — 0″·2, — 0″·3, — 0″·4. I shall call this the method of "progressive corrections", and it was the one which I adopted. In applying it, the rigid method would be as follows:—First to plot the points as actually taken, say in pencil, and then to note the value *in seconds of arc* corresponding to the correction required by each zero, and so to displace each point in the series by its proper interpolated correction *in seconds of arc* as indicated by the adjacent intervals of the diagram. This method presupposes the existence of a certain amount of diagram to indicate about how much 1″ is in different places, and further it is a laborious business, requiring some skill, and liable to gross blunders which are almost impossible to check when it is once done. I preferred therefore to correct the readings of my series in *divisions of the scale, before plotting them*; this involves a slight error when the value of a division at any point differs sensibly from the value near the zero to which it is referred; but we can always keep these corrections small, so that the resultant effect is very slight; and it must be borne in mind that the correction itself depends on the very arbitrary assumption of a perfectly regular disturbance, and, this being so, it hardly seems worth while to burden it with inconvenient ultra-refinements; the arithmetical treatment also is simple and rapid and can be easily checked.

On the opposite page is given one of the series actually taken by me, showing this method of correction. It may be noted that the correction entered against each zero is equal to *half* the difference of its two ends, and has the sign necessary for the right-end reading (the correction to the left-end reading would of course be equal and of opposite sign); similarly the interpolated corrections shown are those to be applied to the right-hand reading. The corrections in this example happen to be very slight, but they exhibit the principle sufficiently well.

The "7" Series. Page 13.

Serial No. of Observation	Micrometer reading of tester	Observed				Corrected for Diagram						Remarks
		No. 6 Level		No. 9 Level		No. 6 Level			No. 9 Level			
		R. end	L. end	R. end	L. end	Correction	R.	L.	Correction	R.	L.	
1	**40**	38·0	38·0	25·9	25·8	**0**	38·0	38·0	**0**	25·9	25·8	Temp. 97·6
2	7	4·5	71·6	−10·8	62·5	·0	4·5	71·6	0	−10·8	62·5	
3	15	12·6	63·6	− 2·3	53·8	+ ·1	12·7	63·5	0	− 2·3	53·8	
4	23	21·3	55·0	6·4	45·2	+ ·1	21·4	54·9	0	6·4	45·2	
5	31	29·2	46·8	15·5	36·1	+ ·1	29·3	46·7	0	15·5	36·1	
6	39	36·7	39·3	24·9	26·8	+ ·2	36·9	39·1	0	24·9	26·8	
7	47	43·9	32·0	34·2	17·3	+ ·2	44·1	31·8	+ ·1	34·3	17·2	
8	55	57·7	18·1	42·9	8·9	+ ·2	57·9	17·9	+ ·1	43·0	8·8	
9	63	66·9	8·8	50·7	0·9	+ ·3	67·2	8·5	+ ·1	50·8	0·8	
10	71	76·9	− 1·3	57·8	− 6·1	+ ·3	77·2	− 1·6	+ ·1	57·9	− 6·2	
11	79	...	...	...	...	...	...	...	+ ·1	...	...	
12	**40**	37·6	38·1	25·6	25·8	**+ ·3**	37·9	37·8	**+ ·1**	25·7	25·7	
13	79	...	...	...	...	...	...	...	+ ·1	...	...	
14	71	76·2	− 0·6	57·7	− 6·1	+ ·3	76·5	− 0·9	+ ·1	57·8	− 6·2	
15	63	66·9	8·5	50·4	1·2	+ ·3	67·2	8·2	+ ·1	50·5	1·1	
16	55	57·6	18·0	42·4	9·2	+ ·3	57·9	17·7	0	42·4	9·2	
17	47	43·4	32·3	33·8	17·8	+ ·2	43·6	32·1	0	33·8	17·8	
18	39	36·6	39·0	24·0	27·4	+ ·2	36·8	38·8	0	24·0	27·4	
19	31	28·9	46·7	14·8	36·9	+ ·2	29·1	46·5	0	14·8	36·9	
20	23	21·0	54·6	5·9	45·6	+ ·1	21·1	54·5	− ·1	5·8	45·7	
21	15	12·5	62·9	− 2·6	54·0	+ ·1	12·6	62·8	− ·1	− 2·7	54·1	
22	7	4·5	71·0	−10·2	61·7	+ ·1	4·6	70·9	− ·1	−10·3	61·8	
23	**40**	37·8	38·0	25·9	25·6	**+ ·1**	37·9	37·9	**− ·1**	25·8	25·7	
24	7	4·1	71·2	−10·1	61·6	+ ·1	4·2	71·1	− ·1	−10·2	61·7	
25	23	20·5	54·9	5·9	45·5	+ ·1	20·6	54·8	− ·1	5·8	45·6	
26	39	36·9	38·5	24·7	26·7	+ ·1	37·0	38·4	− ·1	24·6	26·8	
27	55	57·7	17·6	42·5	9·0	+ ·1	57·8	17·5	− ·2	42·3	9·2	
28	71	76·4	− 1·5	57·7	− 6·2	+ ·1	76·5	− 1·6	− ·2	57·5	− 6·0	
29	63	67·9	7·3	50·3	1·1	+ ·1	68·0	7·2	− ·2	50·1	1·3	
30	47	43·7	31·5	33·5	17·8	+ ·1	43·8	31·4	− ·2	33·3	18·0	
31	31	28·9	46·5	14·6	36·6	+ ·1	29·0	46·4	− ·3	14·3	36·9	
32	15	12·7	62·5	− 3·0	54·2	+ ·1	12·8	62·4	− ·3	− 3·3	54·5	
33	**40**	37·4	37·6	25·9	25·4	**+ ·1**	37·5	37·5	**− ·3**	25·6	25·7	
34	15	12·2	62·8	− 3·0	54·1	+ ·1	12·3	62·7	− ·3	− 3·3	54·4	
35	31	28·8	46·4	14·9	36·5	+ ·1	28·9	46·3	− ·2	14·7	36·7	
36	47	43·5	31·6	34·0	17·5	+ ·1	43·6	31·5	− ·2	33·8	17·7	
37	63	67·3	7·6	50·5	0·9	+ ·1	67·4	7·5	− ·1	50·4	1·0	
38	79	...	...	...	...	...	...	...	− ·1	...	...	
39	71	76·9	− 1·9	57·7	− 6·2	+ ·2	77·1	− 2·1	0	57·7	− 6·2	
40	55	57·7	17·4	42·2	9·1	+ ·2	57·9	17·2	0	42·2	9·1	
41	39	36·6	38·4	24·2	27·1	+ ·2	36·8	38·2	0	24·2	27·1	
42	23	21·0	54·0	5·3	46·0	+ ·2	21·2	53·8	+ ·1	5·4	45·9	
43	7	4·1	70·7	−10·8	62·0	+ ·2	4·3	70·5	+ ·1	−10·7	61·9	
44	**40**	37·2	37·6	25·6	25·7	**+ ·2**	37·4	37·4	**+ ·1**	25·7	25·6	Temp. 98·0

Note.—In actual practice a third column should be given to each level, under the heading "Observed", in which the sum of the two readings, or "length of the bubble", may be entered; as it is only required to roughly check the accuracy of the readings I have omitted it here.

I have gone into this question of zero correction at some length, as it is one on which a fresh experimenter will inevitably spend much thought and time; but it must be remembered that, as a matter of fact, if the number of our observations is large, and if we calibrate every curve, the effect of slight errors will be practically eliminated from the final diagram.

PART IV.—FINAL NOTES.

Suggested Modifications of Procedure.

In Appendices I and II are embodied my proposals for the complete calibration of levels; in Appendix I will be found suggestions for the determination of the zero curve by the observer, after which the bulk of the testing as detailed in Appendix II might be handed over to an assistant. The observer might so obtain "abstract diagrams" of several levels and choose the best, after which he would carry through a little final work on those parts of the chosen level which he proposed to use in actual work, employing every refinement he could devise, as suggested in Appendix I. In Appendix II, I have laid down, for the bulk of the work, my procedure as above described, with the following modifications:—

(1) The zero for every bubble would be that given by the zero curve, previously determined, instead of the assumed position of "equal readings."

(2) In order to give more weight to the zero determinations, every one is to be taken twice, first with the bubble brought to rest from a left to right movement, and then one where the bubble is brought into position from the right.

(3) When the difference between the readings of the two ends, with the micrometer at zero, is more than 2 divisions greater than it should be, *i.e.*, when a correction of more than 1 division to each end is necessitated, the adjusting screw of the level concerned must be used to bring it into a more correct position, and then a fresh zero determination is to be taken for use with the remainder of the series; the object of this is of course to keep the corrections small, a matter to which I did not pay nearly sufficient attention in my first calibration.

(4) Different portions of the micrometer screw will constantly be used.

Comparison of the two Diagrams, with remarks on the inevitable gain in accuracy due to calibration.

There are many points of interest which a careful comparison of my two actual diagrams brings out. Indications of irregularities beyond those already noted in Part II are unfortunately much confused by the artificial straightness of the zeros, especially in the case of No. 6 level which obviously should have a zero line very far from straight; even in No. 6 however the roughest measurement will show how much larger the intervals are to the right of zero than to the left (*e.g.*, compare the distance of the + 30″ curve from zero with that of the − 30″ curve); this explains the fact, which the most cursory test corroborates, that, if we apply the mean value system of interpreting level readings to this level, the result is always in excess of the true arc when the bubble is to the right of the tube, and in defect when it is working near the left end. In No. 9 level on the other hand the right hand end happens to have a slightly greater mean value than the left.

I may here state that, in order to prevent variations in the micrometer screw of the tester from disturbing the concordance of results, *I always used a certain portion, viz., from division 0 to division 80, of one particular revolution of the micrometer, during the whole work;* to get the best results it would of course have been better to have used various portions of the micrometer screw so as to eliminate the effect of any variations it may have; but such variations are certainly less than many of the other uncertainties in the work, and, even if they were not, it was most important in first testing my method to establish the feasibility of calibration, so that I did not want my results further disturbed by micrometer irregularities.

An examination of the evidence of the level diagrams seems to show that there is no great periodic or other error in the particular micrometer revolution employed; for the variations in No. 6 level, if due to the tester, would imply that the micrometer had a larger value from divisions 40 to 80 (*i.e.*, on the right of zero) than from divisions 0 to 40, whereas the evidence of No. 9 level contradicts this. At the same time there are slight indications of variations in value between individual divisions of the micrometer, *e.g.*, we may notice that in No. 9 diagram the curves + 12″ and + 13″ are abnormally close together *throughout their length;* now there is no conceivable condition in a level which could have this effect on its diagram, and, as No. 6 diagram shows signs of the same closeness between these two curves, it seems certain that the movement of the tester from division 52 to 53 was somewhat less than the general mean; we may notice a similar effect between curves + 20 and + 21, and in various other places. Trifles like this may all be eliminated as the method is further refined, but I do not think they can be compared with the errors which the arbitrary assumption of a mean value, for the whole level, is liable to introduce.

I have drawn in thick red lines in both diagrams the "mean value" curves at 10″ intervals; these of course are straight lines, and their divergence from the true curves will give an idea of the errors which may be introduced into the two levels by the employment of a mean value.

The striking concordance of my data, collected during several weeks, proves that the variations from the mean value displayed in my diagrams are in keeping with actual facts; and, this being so, it is evident that all the errors due to these variations are eliminated by calibration.

It is obvious that the advantages secured by calibration are inversely proportional to the regularity of the curve of the tube. Thus it may be noted that with the longer bubbles in No. 9 level there is very little variation in the values, compared with that which may be seen everywhere in No. 6 level; even in No. 9 level, however, the variations in value when using short bubbles are very marked; *e.g.*, compare the distances KL and MN, which both equal 10″, in the diagram of No. 9 level.

Now we may improve and strengthen our diagram as much as we please by amassing further data, and so eliminating the effects of transitory disturbances in testing, but no care and no labour can possibly evolve a mean value which will give anything like a true result, both for the movement from K to L, and likewise for that from M to N. In No. 6 level, where variations of this magnitude are very frequent, the advantages of calibration are everywhere most marked.

I think a careful consideration of the diagrams, and of the conditions under which levels are generally used, will show that accumulative errors are very liable to occur in applying a mean value to a series of level corrections to none of which it may be truly applicable. This is however a somewhat controversial point, and a long dissertation, which would be out of place in these notes, would be required to deal with it fully; so I shall content myself by drawing attention to what seems to me to be a very important consideration.

The various qualities of levels.

In addition to the qualities of a level shown in a diagram, there are others which may be noted in testing, and which apparently depend on the nature of the liquid with which the level is loaded, and on other such-like causes. The difficulty of discussing these qualities is much enhanced by the indiscriminate manner in which such epithets as sensitiveness, delicacy and refinement appear to be used. Definite conventions as to the use of words are very necessary for the general crystallization of ideas, and cannot fail to assist in their elucidation and discussion; so, as I cannot find any special conventions with regard to levels, I will take this opportunity of suggesting the following:—

(1) "Delicacy" is a measure of the flatness of the curve, *i.e.*, it is proportional to the radius of curvature, and varies inversely as the mean value of a division (in cases where the divisions of the scales of the levels compared are of equal length); it is evident that the delicacy, as above defined, of an irregular tube will vary at different parts of the tube, but roughly we may say that the two levels examined by me were both very "delicate", and about equal in this respect.[*]

(2) "Sensitiveness" is that quality of a level which causes it to at once respond to the slighest disturbance. In levels of equal delicacy it apparently depends on the nature of the liquid; as No. 6 level containing chloroform is far more sensitive than No. 9 which is loaded with spirit. No. 6 not only indicates the slightest disturbance promptly, but it settles down to its new position of equilibrium long before No. 9 has ceased to move; so that "sensitiveness" would appear to indicate a comparative absence of inertia or skin friction.[†]

(3) I would apply the term "determinateness" to that quality by virtue of which a bubble invariably takes up exactly the same position under the influence of the same dislevelment; it would be indicated by the accordance of the dots plotted in preparing a diagram, and the result of great determinateness in a level would be to decrease the probable error of all its determinations, supposing a diagram to be used. It is obviously a most important quality, and possibly depends on the absence of minute irregularities in the tube, together with the presence of a sufficiently definite curvature (for a straight tube would of course be absolutely indeterminate). No. 9 level is on the whole more determinate than No. 6; but No. 6 is fairly good if we except the bad portions of its tube. Both levels become very indeterminate if we use too short a bubble, but bubbles of 3 inches length and upwards seem sufficiently determinate for ordinary work.

To sum up, we may then say that my two levels are of about equal delicacy, and that No. 6 is far the more sensitive and prompt, while No. 9 is on the whole the more determinate.

If we are applying the "mean value" method to a level, there is another quality which becomes of paramount importance, *viz.*—regularity of the curve, *i.e.*, uniform "delicacy" throughout the length of the tube; but this quality seems practically unattainable, and if a level is properly calibrated we care little for variations of curvature within reasonable limits as long as delicacy and determinateness are attained over the greater portion of the tube.

The above conventions are tolerably accordant with the accepted meaning of the terms employed, and their general adoption would simplify the description of any individual level, and would eliminate much of the vagueness so generally noticeable in discussions of this subject. I

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[*] This term is chiefly intended as a rough general description of a level, and is so used throughout these notes.

[†] Should further investigation show that these two qualities are not necessarily concomitant, the word "promptitude" would be a sufficiently obvious description of the quality of quickly settling.

think too that we may look for improvement in the manufacture of levels once we are able to point out to the makers the exact faults of those we possess, and can by experiment answer such questions as:—"How far may the delicacy and sensitiveness of levels be increased without producing indeterminateness?" "What is the comparative sensitiveness of various liquids?" "Does extreme sensitiveness in the liquid militate against determinateness?".*

As far as I am aware there is at present very little definite information to be had on such questions as these; yet they are all open to direct solution by experiment, and such information is very requisite as a preliminary to real progress in improving the level as an instrument.

Considerations regarding the general adoption of calibration.

I cannot claim to be a perfecty qualified or unprejudiced judge of this matter; so I shall briefly state the case for calibration, as it appears to me, and leave the full consideration of the *pros* and *cons* to impartial observers of greater experience.

In the first place, I confidently rely on the figures in Appendix III to show that the diagrams are a more accurate means of interpreting level corrections than any mean value. It has, however, been pointed out to me that the unreliability of the "mean value" system is fully recognized, and that ample precautions are taken to balance all sources of error which may be due to it; so that it still remains a question whether the amount of labour, entailed in calibrating, will be repaid by any appreciable increase of accuracy in ordinary work, where a large number of observations are massed together to obtain the final mean.

This argument, however true it may be, —— and I must say I cannot quite subscribe to it myself —— does, at least, admit the superiority of calibration in dealing with experiments of great refinement, or of an especial nature where the variations of a quantity are quite as important as the final mean; such as observations of latitude variation. In addition to this, I think the power given by the diagrams in discriminating between different levels, and in avoiding the more indeterminate portions of them, and, generally, in investigating the whole subject, cannot be denied.

I shall therefore assume that the above advantages of calibration are beyond controversy; so it only remains for me to adduce such arguments as I can in favour of adopting the method in dealing practically with the great mass of ordinary field observatory work.

First, as regards the amount of work required by my scheme, it must be remembered that my present proposals aim at a complete examination of the levels with a view to elucidating the laws which govern their vagaries. If an observer chooses to maintain a constant length of bubble, within small limits, throughout his work, and to always keep his bubble near any particular portion of the tube, his calibration work will be correspondingly reduced, and a very narrow short strip of diagram, which could be made in a few hours, will meet all his practical requirements.

Even for work of secondary refinement, where the examination of the levels is liable to be more or less cursory, I would point out that the mere fact of plotting the points on a diagram would keep before the mind of the observer the actual scantiness of his investigation, and the magnitude of his assumption in applying a mean value to the whole level on the strength of it.

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* The possibility of this is suggested by the qualities shown by my two levels; it might be due to the small inertia (which "sensitiveness" implies) being insufficient to overcome increased capillary or other resistance caused by microscopic faults in the tube, whereas a liquid having more inertia might be able to insist on taking up its determinate position in spite of such trifling impediments. I am further inclined to think that a high co-efficient of expansion may be objectionable, as I fancy that when a stationary bubble changes its length it is liable to receive a greater increment at one end than at the other, the inertia of the liquid preventing a proper readjustment until the bubble is set moving again. The amount of the variations in length can only be decreased by having a liquid of low co-efficient of expansion or by ***increasing the volume of the bubble as compared with that of the liquid.***

Further, even a few points plotted might suffice to show up any large irregularities in the level, (such as a greater flatness on one side of the centre than on the other); so that, even if the majority of the curves of a diagram were filled in by interpolation amongst such points, the result would obviously be more in keeping with the actual facts than the assumption of a mean value.

It is, however, of the first importance to practical observers to have some assurance of the *constant applicability* of the diagrams through long periods of time; as, though they might be willing to undertake the labour of a very complete calibration if likely to be of permanent value, it would generally be quite impracticable to repeat the whole process every few months. This is, I think the crucial point on which the whole question hangs; it can only be definitely settled by testing the diagrams of a *large number** of levels after a considerable lapse of time. Owing to the change of level scales on the completion of my first calibration, and to the fact that the tubes have been dismounted and altogether readjusted, and that true zero curves are being employed, instead of "straight-line" zeros, in the new calibration, the comparison of it, when completed, with my former work, can hardly give very conclusive results. At the same time, all the evidence I have been able to collect shows clearly that the main features, at all events, of the tubes, have remained constant throughout an interval of one or two years; for instance, the agreement, noted in Appendix I, of the zero curves, which I have recently deduced, with the indications of the old diagrams, is a very satisfactory piece of evidence on this point, and I have gathered a good deal more of a still more definite nature. So that, although I feel that dogmatic statements, even regarding my own two levels, must be postponed, until my new diagrams can be re-examined after a sufficient lapse of time, still I am personally very confident that the result will be satisfactory. Even if it should be found that slow changes of any kind are liable to occur, —— and I feel sure that this is the worst that need be apprehended —— the diagrams will still remain for some time, probably several years, a more accurate approximation to actual facts than any mean value; and, further, a narrow short strip of diagram could easily be re-determined to meet practical requirements.

I have not here attempted to deal with, or criticise, the many expedients and precautions adopted by modern observers to minimise the effects of irregularity in the level tubes, but have contented myself with frankly advocating my own system. It would be pretentious and tedious for me to attempt to go into the whole question at length, and my opinion can have little weight except regarding my own particular work. Also my opportunities for enquiring into previous researches in this subject have been very limited, and I may have occasionally treated as discoveries matters of which more experienced observers are well aware.

There would seem to be little doubt, however, that my system is in the main quite new; and, imperfect as it may be in its present experimental stage, I trust I have made its possibilities sufficiently clear; and I cannot but hope that, if the matter were taken up and developed by others more competent and better equipped than myself, great improvements might be effected in the use of the level as an astronomical instrument.

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* For it is quite conceivable that the nature of the glass or of the liquid in the level may affect this question; so that one is hardly justified in jumping to conclusions, from the results, however satisfactory, in one or two individual cases.

APPENDIX I.

Practical hints and suggestions for further refinements in procedure, with notes on the determination of the zero curve.

Preliminary.

The whole procedure here described is only applicable to levels provided with reservoirs, or air-chambers, by means of which the length of bubble can be altered at will. With levels not so provided, the difficulties of calibration would be much enhanced, and the procedure must be modified accordingly.

I think it is generally very desirable, throughout the testing, to keep the levels in the cells or cases which carry them when mounted on the instrument. These cases protect the levels from petty fluctuations of temperature; also it may be possible to fix diminutive cross levels on them, by means of which we can make sure that the cross-levelling during testing is the same as that when mounted for work; I am confident that this precaution is very necessary with some tubes.

Before commencing calibration every care should be taken to see that the tube is satisfactorily mounted in its case, and that the scale, if movable, is of a satisfactory pattern and is conveniently mounted in a proper position. Once the work is commenced no alteration of these details should be attempted.

I have recently mounted my levels with sloping ivory scales whose lower edge is forward of the centre of the tube; so that, when the observer looks down on the level at an angle of 45°, the lower edge of the scale intersects the centre of the bubble. This facilitates an accurate reading, but gives considerable parallax, owing to the edge of the scale being a good deal forward of the centre of the bubble. To eliminate the effects of this parallax, some device was necessary to regulate the position of the eye in reading; so I have mounted each case with a long strip of mirror, about half an inch wide, parallel with the scale, and appearing just below the bubble. This mirror is sloping, so that the reflection of the eye at 45° can be seen in it; and I make it a rule to glance at the mirror before taking a reading, and to move my eye until its reflection is seen near the upper edge of the mirror and just below the end of the bubble to be read. I have found this arrangement very satisfactory, and it gives no trouble in practice, provided that the mirror is given a convenient slope.*

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* Working in a dark observatory, it is often difficult to find the reflection of the eye, so I use a spill of white paper which I hold vertically under my eye against my cheek bone; the reflection of this is easily caught, and when its upper end is seen at the lower edge of the mirror, one knows that one's eye must be reflected from pretty near top edge.

Having made all preliminary adjustments, it is important to note the *exact position of the zero of the scale* on the tube, and also the lineal value of the divisions of the scale, in case of its ever having to be replaced; it is well to mark the tube in such a way that, if it has to be re-adjusted, it may be possible to replace it in its case *exactly* as before.

General Scheme.

It is suggested that the observer should himself first determine the zero curve of a level, and take note of its general qualities, such as sensitiveness, promptitude, etc. He would then hand over the bulk of the work, detailed in Appendix II, to an assistant, who might so do preliminary sheets of several levels. On receipt of these, the observer could in a few hours make out the abstract diagrams, which would enable him to decide on and choose the best level, and also to determine what portions of it, if any, should be avoided on account of faults. He might finally re-determine such portions of the diagram as he required for actual work, using every possible refinement, and arriving at a good idea of the real probable error and peculiarities of the instrument.

There is no reason why two or more levels should not be tested simultaneously; it is certainly advantageous to have more than one, to assist in defining the cause of any occasional abnormal result, which may have been simply due to a movement of the pillar or supports which carry the levels.

Determination of the zero curve.

I have explained in Part II that the essential object of this portion of the work is as follows:—

Having chosen any given position of the level-tube as the zero position, we wish to discover the positions which bubbles of different lengths will take up when the tube is in that position; so that, whatever the length of the bubble may be, we may be able at once to bring the tube into the zero position.

This ensures that all the zeros of different bubbles shall correspond to one and the same identical position of the tube, and gets rid of the inaccurate assumption involved in taking the "equal-readings" position of *every* bubble as its zero position.

It is convenient to choose the "equal-readings" position of *some particular bubble* (preferably a good long one) as the zero position of the tube; then, it is required to find the positions of all other bubbles corresponding to that position. By so doing it is evident that our ultimate diagram will be a differential calibration referred to the "equal-readings" position of this chosen bubble. I shall call this the "reference" bubble, and its length should always be noted on the diagram, so that all future determinations of the zero curve may likewise be referred to it.

As it is impossible to change the length of a bubble at will without disturbing the level, we require some means of replacing the level exactly in its former position, after having lifted and tilted it to alter the length of the bubble. Considering the delicacy required, I think it would be impossible to effect this by any purely mechanical precautions. If, however, we attach the level to be examined to an auxiliary level *which has no reservoir*, we may lift up the case containing the two and tilt it about as we please to obtain any required length of bubble in the real level, *without altering the bubble in the auxiliary level;* then, if the mechanical rigidity of the connection between the two levels is absolutely reliable, we have only to replace the case on an adjustible stand, and bring the bubble of the auxiliary level to its former position, and we know that *both tubes* must be exactly as before; so that a comparison of the positions of the bubbles of the *real* level before and after the operation will afford the data required. A uniform temperature throughout the experiment is very desirable, so that the bubble of the auxiliary level may remain quite unchanged.

I have had a simple apparatus made to carry out this experiment; and, as its purpose is the determination of the zero curve, I am calling it a "zerometer".

The zerometer consists essentially of an adjustible support carrying a removable upper plate or tray, to which the levels are fixed.

The removable tray is a thick oblong plate of brass having a handle at each end; it stands on three small fixed feet which are carried by suitable **V**'s on the supporting plate. On the tray is fixed the auxiliary level; this auxiliary level is adjustible with regard to the tray, but can be rigidly clamped when the required position has been attained; the tray also bears an arrangement by which any other level to be examined can be rigidly fixed to it, the two levels naturally lying parallel to one another.

The adjustible support is a solid base-plate shaped roughly like an isosceles triangle; it stands on three footscrews, two at one end, and one (which should be a fine screw) at the other; it carries three **V**'s to take the feet of the tray, and is fitted with a small cross level.

Let us, for brevity, call the level to be tested "the level T", and the permanent Zerometer level "the level Z". The procedure is then as follows:—

(1) Set up the zerometer.
(2) Fix the level T rigidly to the tray.
(3) By means of the end footscrew of *the base-plate,* bring the bubble of T to a central position.
(4) Then, without disturbing the tray, bring the bubble of Z, which is adjustible with regard to the tray, to a central position.
(5) Clamp the level Z; after this do not touch either level, so that their mutual position may remain absolutely unchanged throughout the work.
(6) Pick up the tray by the handles, and, by tilting it about as required, adjust the length the bubble of T until it is within a division or two of that which we have decided to use as a "reference bubble".
(7) Replace the tray, and bring the reference bubble of T to equal-readings by means of the zerometer footscrew. The level T is now in its zero position, and it only remains to note the corresponding position of the bubble of Z, which position will be the *reference zero* of Z during the experiment.
(8) Now take up the tray and give T any required length of bubble; then, after replacing the tray, adjust the zerometer by its footscrew until the bubble of Z is at its reference zero. *This brings the tube of T to its zero position,* so the resultant position of the bubble of T is the required zero of that particular bubble.
(9) This observation may be repeated with successive bubbles of various lengths in T, until we have all the data required for plotting our zero curve, which will henceforth show the exact zero position of any given bubble.

I have found this experiment wonderfully simple and satisfactory; the results turned out so accordant that it was possible to completely determine a long zero curve for each level by two or three hours of observing.

In arranging the lengths of bubble required, it is best to commence from a short bubble and to increase it by small successive increments, in preference to trying to shorten down from a long bubble; in the former case it will be found that by holding the tray at a critical tilt and tapping the lower end with the forefinger, very small bubbles can be liberated from the air-chamber at will, so that after a little practice the bubble can be easily made the required length within a division or two; whereas in shortening the bubble it is almost impossible to see what is happening.

The experiment is slightly complicated by the fact that we can in no case ensure getting the bubbles to the exact positions required, so that small corrections have to be introduced throughout to reduce the readings to those positions, *e.g.*, in para. 7 above, the reference bubble of T will probably not be exactly at equal readings, so the observed reference zero of Z will have to be corrected accordingly; similarly in the remainder of the work, the bubble Z can only be approximated to its reference zero position, and the positions of the bubbles of T have to be correspondingly corrected. These corrections should be kept very small, and may be made roughly in divisions if the levels are of about equal delicacy.

In my experiments I took 16 different bubbles of T as one series, and determined the reference zero of Z at the beginning and end of this series, applying the mean of the two results throughout the series.

The zerometer made for me at the Survey of India Offices in Calcutta was so excellently designed, and constructed, that, not only did the two levels maintain their relative position within a second, and often less, throughout the series, but the tray when replaced nearly always reassumed its original position exactly, so that I was very seldom delayed by having to adjust the zerometer to bring the bubble of Z to its reference zero. This result was entirely beyond my expectations, and it shortened and simplified the work very considerably.

In making every determination I took the mean of two observations; in the first of these the bubbles were read after having been brought into position from the left end of the tube, and in the second the bubbles were brought up from the right end. This was to balance any effects of stickiness, and is, I think, a very necessary precaution; it was quite simply effected by slightly raising the required end of the tray and gently replacing it, the result of course being that the bubbles, having rushed to the raised end of their tubes, had to move into position from that end.

I may note in passing that the whole of this experiment would be a most satisfactory means of testing whether the mounting of the level in its case or cell is rigid and reliable; I have shown how accordant the results from this zerometer may be when the mounting is good.

Further, if we have calibrated a diagram to a true zero, a very interesting examination of any given curve could be made on the zerometer; for, having taken the position of some particular bubble on that curve as the reference zero, we could see if all other bubbles, referred to that zero by the zerometer, consistently took up positions on that curve. Such an experiment, if successful, would afford most irrefutable evidence of the quality and reliability of a diagram.

The interpretation of the zero curve.

I have already shown, in Part II, that one great advantage of discovering the zero curve is that, if it happens to deviate considerably from a straight line, the use of it avoids confusing distortions of the whole diagram, which would arise if an assumed straight zero were employed; further, I have stated that, by calibrating to a true zero we have some reasonable hypothesis to assist us in drawing in the other curves, and adjusting any conflicting data which may occur in the "abstract" diagram. I think it may be well to now repeat more explicitly the above-mentioned "reasonable hypothesis."

The theory, as deduced from the considerations detailed in Part II, is shortly as follows:—
Let us assume that we have found a zero curve, as shown in red in figs. 9 and 10, and having a fault *ab*; let the readings of the positions *a* and *b* be as follows:—

a, right-hand reading 35, left-hand reading 40,
b, " 30, " 30,

then I conclude that the fault *ab* is due either to a steep bit of curve between the right-hand readings 30 and 35, or to a flat bit between the left-hand readings 30 and 40; (it might of course be conceivably due to a combination of both these causes). I cannot tell which of these explanations is the true one until I have worked out some more of the curves on the diagram; but if

Fig. 9

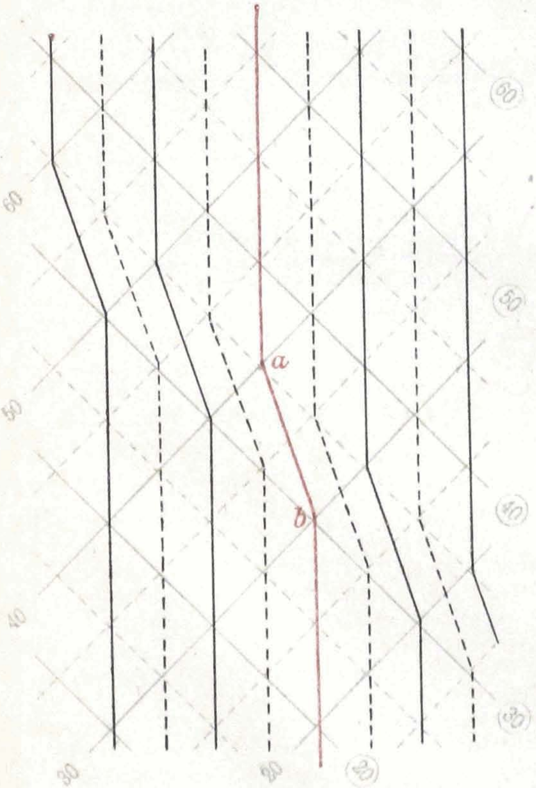
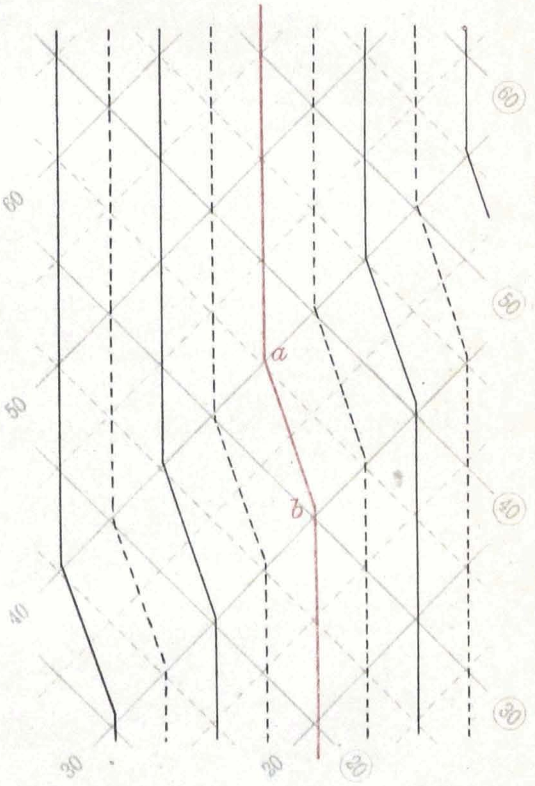


Fig. 10



the fault is due to a steep bit between R 30 and R 35, I expect to find the curves as shown in fig. 9; and if it is due to flatness between L 30 and L 40, I expect curves as shown on fig. 10.

In any actual diagram the matter is likely to be complicated by the presence of many small irregularities in the tube; but I think a consideration of these two figures will furnish a sufficient idea of the sort of relation between different curves which may be expected.

In conclusion, it may be interesting to note that the zero curves which I have just determined for my two levels are in complete accordance with my expectations.

In No. 9 level the zero curve turns out to be nearly straight, but has a slight, though perfectly distinct, drift towards the left as the bubble gets longer; this is due to the fact, previously pointed out, that the right-hand side of No. 9 level has a slightly steeper curve than the left-hand side.

The chief feature of the zero curve of No. 6 is that all the central portion, and especially that just beyond the 76-division bubble-length, is two or three divisions to the left of equal-readings. Its curvature is about equal, though opposite in direction, to that of the $-10''$ and $-20''$ curves in the diagram on Plate III. This is a further corroboration of the steep bit of curve followed by the flat portion on the right-hand side of this level, which I have already discussed in Part II. Also we see that, as we should expect from a consideration of fig. 8, the general bulge of all the curves in the diagram of No. 6 towards the right is chiefly due to the artificially straight zero to which this diagram has been referred.

APPENDIX II.

Practical hints for carrying out the bulk of the work.*

The preliminary details noted in Appendix I having been attended to, and the zero curve determined, we assume that the observer is provided with a bubble tester, of the pattern described in Part I of the Notes, mounted on a properly isolated pillar, and that he is to test two levels simultaneously.

It must first be decided, with regard to each level:—

(1) The limiting lengths of bubble to be tested. It will be well, in deciding this, to include shorter bubbles than we really want for actual work, as this gives depth to the diagram; and, though somewhat indeterminate, the short bubbles are very useful for showing up irregularities in the tube.

(2) The limiting arcs on either side of zero to be included in the series. For a complete calibration these will be equal to the limits of play of the shorter bubbles. The arc of 40″ on either side of zero, which I have used, will generally cover all requirements.

(3) The frequency of series in each set, *i.e.*, how many determinations are required on every curve; I think determinations on successive bubbles varying 5 or 6 divisions in length will be found to give very ample data.

(4) What pause is required after moving the bubbles, to allow them to settle, before reading. This must be decided by experiment, and ample margin should be allowed. It will probably be found that a longer interval is required for short bubbles than for long ones. A pause of 30 or 40 seconds is sufficient for the least prompt of my levels.

(5) Assuming that we are to calibrate to a true zero, we require a curve plotted on section paper so as to show the right-hand reading of every possible length of bubble to be tested. The observer at every zero requires to know what the zero position of his bubble should be, and it is more convenient for him to find this from such a curve than from the zero curve plotted on the diagram; for the recorder can at once inform him of the length of his bubble, and then a glance at the curve shows him whether the right end of his bubble is within a division of its proper position. Such a curve is of course easily deduced from the proper zero curve of the level.

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* These have been written for the guidance of an untrained assistant in the Survey Department, so that some of the precautions detailed may seem very superfluous to observers of experience.

Having settled these preliminary points, set up the bubble-tester with the micrometer to the observer's right; see that the screws are clean and well oiled, and that the arrangement for taking the weight of the hand is so adjusted that there may be no danger of disturbing the instrument in turning the micrometer.

Observing.

(1) Having given the levels any required lengths of bubble, place them on the **V**'s, and see that their cross-levelling is correct.

(2) Bring the micrometer to the zero position; then, for each level separately, note the length of the bubble and see, from the curve described in para. 5 above, what the corresponding right-hand reading should be; bring the bubble to this position as nearly as possible, by means of its own adjusting screw, and without disturbing the micrometer from its zoro position.

(3) Throw the bubbles to the *left* by turning the micrometer, and then return *it* to its zero position; then, after allowing the proper pause, (a watch should always be used to ensure this) read the bubbles. Then commence by throwing the bubbles to the *right*, and repeat the remainder of the observation as before.

(4) The mean of these two observations will give the first zero determination; and every succeeding zero will be similarly determined by two observations, the first from a left to right movement of the bubbles and the second from a right to left movement. After taking each zero, the observer will note, before proceeding, whether the right end of each bubble is within a division of its proper position; if, in either level, this is not the case, he will re-adjust the level as required, and make a fresh zero determination after adjustment. The zero determination before adjustment will then be available for obtaining the interpolated corrections of previous observations, and the determination after adjustment will be used for all subsequent corrections.*

(5) All the series will be as described in Part III, except that every zero determination will be a double observation as above described; also the numbers of the series will be varied, from time to time, by adding a constant to every number in the series; so that different portions of the micrometer screw may be brought into play.

(6) The recorder will have the micrometer readings, as required for the series, all written out beforehand, and, on the completion of each observation, he will inform the observer of the next micrometer position required.

(7) The temperature should be noted at the beginning and end of each series.

(8) In order to see quickly what has been done, and exactly what bubble-lengths remain to be determined in each set, a "record slip" must be kept for each level. A strip of section paper is convenient for this purpose; on it the vertical columns may be marked off to show all the lengths of bubble to be tested, and the depth for each level will be eight horizontal rows, one for each set. Then, as soon as a series has been completed, the recorder can, in each level, mark off the square corresponding to the proper set and the length of bubble used. This enables the progress of the work to be seen at a glance, and is a very great convenience.

(9) The computation, plotting, and abstraction will be carried out as detailed in Part III.

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* It is assumed that the method of "progressive corrections," described in Part III of the Notes, is to be adopted.

APPENDIX III.

Tables shewing the respective Values of Arcs obtained by the use of (1) the Diagram and (2) the Mean Value.

In the following Tables are embodied the results of test observations taken in Dehra Dún in July 1900. In carrying them out the procedure was as follows:—The two levels No. 6 and No. 9 were carefully placed in the V's of the Bubble Tester,—due care being given to the cross-levelling. Various readings of the Micrometer Head of the Bubble Tester were taken and recorded together with the corresponding readings of both the Bubbles. From the differences of the bubble readings, the arcs moved through by the Bubbles were computed, using a mean value. The same arcs were then determined from the Diagram of 1899 and also from the Diagram of 1900. These three values for each arc are entered in the Tables and compared with the values as given by the Bubble Tester. As will be seen from the Tables, Bubbles of various lengths were tested.

HOLMES' LEVEL No. 6

Mean Value of 1 Division of Scale = 0″·8635.

Reading of the Micrometer of the Bubble Tester	Bubble Reading		Value of the Arc in Seconds			
	Right Hand	Left Hand	As given by the Bubble Tester	As given by the Diagram of 1899	As given by the Diagram of 1900	As given by using a mean value
			″	″	″	″
40	32·7	33·4	...	...	...	...
45	37·6	28·4	5·0	4·6	4·6	4·3
50	43·8	22·2	5·0	6·1	5·1	5·4
55	51·7	14·3	5·0	4·4	4·5	6·8
60	59·2	6·3	5·0	5·2	4·8	6·7
65	63·6	2·1	5·0	4·5	5·1	3·7
63	61·9	3·8	...	...	...	...
58	55·9	9·8	5·0	5·4	...	5·2
53	47·9	17·9	5·0	4·4	5·4	6·9
48	39·9	25·8	5·0	6·3	4·6	6·9
43	35·1	30·3	5·0	4·0	4·5	4·0
38	29·4	36·2	5·0	6·0	4·9	5·0
30	20·8	44·9	...	...	...	...
40	31·0	34·7	10·0	10·5	9·1	8·8
45	36·4	29·1	5·0	5·3	5·2	4·8
35	26·6	39·1	10·0	10·2	9·5	8·6
5	−7·7	73·1	...	...	...	...
25	14·9	50·8	20·0	22·3	...	19·4
40	31·6	33·7	15·0	16·4	14·9	14·6
20	9·0	56·1	20·0	20·9	...	19·4
40	−2·3	74·4	...	...	...	...
45	4·0	67·9	5·0	6·2	6·6	5·5
50	9·9	61·8	5·0	5·2	4·1	5·2
55	16·0	55·7	5·0	5·5	4·6	5·3
60	21·8	49·8	5·0	5·7	...	5·1

HOLMES' LEVEL No. 9

Mean Value of 1 Division of Scale = 0″·9346.

Reading of the Micrometer of the Bubble Tester	Bubble Reading		Value of the Arc in Seconds			
	Right Hand	Left Hand	As given by the Bubble Tester	As given by the Diagram of 1899	As given by the Diagram of 1900	As given by using a mean value
			″	″	″	″
40	31·7	31·6	...	...	...	...
45	37·4	25·8	5·0	5·6	...	5·4
50	43·0	20·1	5·0	5·3	...	5·3
55	48·5	14·5	5·0	5·0	5·5	5·2
60	53·9	8·9	5·0	5·4	...	5·1
65	59·3	3·5	5·0	5·2	...	5·0
63	57·0	5·9	...	...	...	...
58	51·5	11·5	5·0	5·3	5·2	5·2
53	45·9	17·2	5·0	5·4	...	5·3
48	40·6	22·4	5·0	5·2	...	4·9
43	34·9	28·1	5·0	5·8	5·7	5·3
38	29·8	33·3	5·0	4·4	4·5	4·8
30	25·7	40·2	...	...	...	...
40	35·8	29·9	10·0	9·0	9·1	9·5
45	41·6	24·3	5·0	5·7	5·5	5·3
35	30·9	34·8	10·0	10·3	10·0	9·9
5	−0·6	66·4	...	...	...	...
25	20·0	45·5	20·0	19·2	17·8	19·4
40	36·0	29·7	15·0	14·5	14·9	14·9
20	15·0	50·7	20·0	19·1	19·4	19·6
40	8·8	61·3	...	...		...
45	15·5	55·7	5·0	5·1		5·8
50	20·3	49·8	5·0	4·7		5·0
55	25·9	44·2	5·0	5·6		5·2
60	31·1	39·2	5·0	5·0		4·8

HOLMES' LEVEL No. 6

Mean Value of 1 Division of Scale = 0″·8635.

HOLMES' LEVEL No. 9

Mean Value of 1 Division of Scale = 0″·9346.

	HOLMES' LEVEL No. 6						HOLMES' LEVEL No. 9					
Reading of the Micrometer of the Bubble Tester	Bubble Reading		Value of the Arc in Seconds				Bubble Reading		Value of the Arc in Seconds			
	Right Hand	Left Hand	As given by the Bubble Tester	As given by the Diagram of 1899	As given by the Diagram of 1900	As given by using a mean value	Right Hand	Left Hand	As given by the Bubble Tester	As given by the Diagram of 1899	As given by the Diagram of 1900	As given by using a mean value
			″	″	″	″			″	″		″
65	27·4	44·1	5·0	5·5	…	4·9	36·7	33·6	5·0	6·1	These Bubble Readings are not capable of interpretation by the Diagram of 1900, this Diagram being incomplete.	5·2
70	33·3	38·8	5·0	6·0	5·1	4·8	41·6	28·6	5·0	4·0		4·6
68	31·0	40·8	…	…	…	…	38·9	31·4	…	…		…
63	25·2	46·3	5·0	5·9	…	4·9	34·5	35·9	5·0	4·6		4·2
58	19·8	52·0	5·0	5·5	5·2	4·8	28·8	41·5	5·0	5·7		5·3
53	14·1	57·9	5·0	5·3	4·6	5·0	23·5	46·8	5·0	4·9		5·0
48	7·6	64·9	5·0	6·3	5·6	5·8	18·0	52·2	5·0	5·0		5·1
40	−2·3	74·1	…	…	…	…	9·4	61·1	…	…		…
50	9·8	61·8	10·0	11·3	10·5	10·5	20·3	49·8	10·0	9·5		10·4
45	4·0	68·0	5·0	5·2	4·0	5·2	15·0	55·2	5·0	4·5		5·0
35	−7·9	79·6	10·0	…	11·5	10·1	3·1	67·2	10·0	10·9		11·2
45	37·1	33·0	…	…	…	…	54·5	16·5	…	…		…
40	32·7	37·9	5·0	4·4	…	4·0	49·5	21·5	5·0	4·3		4·7
20	10·0	60·2	20·0	22·0	20·4	19·4	28·7	42·3	20·0	20·1		19·4
0	−11·6	81·8	20·0	…	…	18·7	7·9	63·1	20·0	19·0		19·4
45	4·1	70·4	…	…	…	…	14·4	60·9	…	…		…
50	9·4	65·4	5·0	5·0	4·2	4·4	20·1	55·1	5·0	5·0		5·4
55	15·9	58·7	5·0	6·1	5·8	5·7	26·0	48·9	5·0	4·2		5·7
60	21·6	53·1	5·0	5·6	4·6	4·9	31·4	43·8	5·0	5·1		4·9
65	27·2	47·1	5·0	5·5	5·6	5·0	36·6	38·4	5·0	5·2		5·0
70	33·0	41·4	5·0	6·0	…	5·0	42·4	32·6	5·0	5·6		5·4
68	30·5	43·7	…	…	…	…	40·1	34·8	…	…		…
63	25·2	49·1	5·0	5·9	…	4·6	34·8	40·1	5·0	5·2		5·0
58	19·8	54·6	5·0	5·5	…	4·7	29·7	45·3	5·0	5·4		4·8
53	14·1	60·2	5·0	5·0	5·0	4·9	24·4	50·8	5·0	4·8		5·0
48	6·9	67·6	5·0	6·2	5·1	6·3	18·7	56·3	5·0	5·6		5·2
43	0·9	73·7	5·0	7·0	5·0	6·2	13·1	61·8	5·0	5·0		5·2
38	−5·4	79·7	5·0	…	…	5·3	7·0	68·1	5·0	5·2		5·8
40	14·0	60·1	…	…	…	…	27·0	48·0	…	…		…
50	25·4	48·6	10·0	10·7	10·6	9·9	37·3	37·7	10·0	10·1		9·6
45	20·0	54·1	5·0	5·2	5·3	4·7	31·9	43·2	5·0	5·2		5·1
35	7·3	67·0	10·0	11·4	10·3	11·6	21·8	53·1	10·0	9·9		9·4
30	7·7	73·8	…	…		…	…	…	…			…
35	12·9	68·1	5·0	4·7		4·7	31·6	52·2	…			…
40	19·3	61·8	5·0	7·2		5·5	36·8	47·2	5·0			4·8
45	24·9	56·2	5·0	6·9		4·8	41·7	42·2	5·0			4·6

HOLMES' LEVEL No. 6

Mean Value of 1 Division of Scale = 0″·8635.

Reading of the Micrometer of the Bubble Tester	Bubble Reading		Value of the Arc in Seconds			
	Right Hand	Left Hand	As given by the Bubble Tester	As given by the Diagram of 1899	As given by the Diagram of 1900	As given by using a mean value
			″	″	″	″
50	30·2	50·7	5·0	5·7	These Bubble Readings are not given by the Diagram of 1900.	4·7
55	35·2	45·8	5·0	4·9		4·3
53	33·1	48·0	...	...		...
48	27·8	53·1	5·0	6·3		4·5
43	22·2	58·9	5·0	5·4		4·9
38	16·5	64·3	5·0	5·2		4·8
33	10·0	70·9	5·0	6·1		5·7
28	4·3	76·8	5·0	5·9		5·0
55	44·4	36·3	...	...		...
45	33·7	47·1	10·0	11·9		9·3
35	23·0	57·3	10·0	11·4		9·0
25	11·5	69·2	10·0	11·1		10·1
40	29·0	51·4	...	...		...
60	51·8	28·9	20·0	21·4		19·6
70	64·3	16·1	10·0	11·7		10·9
50	38·2	42·2	20·0	21·3		22·5
25	22·4	51·2	...	...	...	...
45	43·0	30·7	20·0	21·7	...	17·7
40	38·0	35·7	5·0	5·9	5·0	4·3
20	17·1	56·4	20·0	21·6	20·3	18·0
35	14·0	69·1	...	...		...
40	20·0	62·8	5·0	6·1		5·3
45	26·0	57·1	5·0	6·3		5·1
50	30·8	52·1	5·0	5·2		4·2
55	36·2	46·7	5·0	5·1		4·7
53	35·0	48·0	...	...		...
48	29·1	53·7	5·0	6·3		5·0
43	23·9	59·1	5·0	5·5		4·6
38	18·0	65·2	5·0	6·0		5·2
33	12·3	70·7	5·0	4·8		4·8
28	5·7	77·3	5·0	6·3		5·7
50	38·4	44·3	...	...		...
60	51·0	32·1	10·0	11·4		10·7
55	45·8	39·4	5·0	5·5		6·3
45	33·3	49·9	10·0	11·7		9·1

HOLMES' LEVEL No. 9

Mean Value of 1 Division of Scale = 0″·9346.

Bubble Reading		Value of the Arc in Seconds			
Right Hand	Left Hand	As given by the Bubble Tester	As given by the Diagram of 1899	As given by the Diagram of 1900	As given by using a mean value
		″	″		″
47·0	37·1	5·0	The Diagram does not include these Bubble Readings.		4·9
52·1	31·6	5·0			5·0
50·0	34·0	...			...
44·8	39·2	5·0			4·9
39·5	44·5	5·0			5·0
34·8	49·2	5·0			4·4
29·4	54·5	5·0			5·0
24·0	59·7	5·0			5·0
51·4	28·4	...	...		...
41·1	38·6	10·0	9·9		9·6
31·1	48·5	10·0	9·3		9·3
20·7	58·8	10·0	9·8		9·7
36·6	43·2	...	...		...
57·2	22·4	20·0	19·6		19·3
...	...	...	...		...
46·6	33·1	10·0	...		10·0
37·2	37·4	...	...		...
57·8	17·1	20·0	20·1		19·1
52·6	22·2	5·0	5·4		4·8
32·6	42·2	20·0	19·0		18·7
27·0	52·8	...	...		...
31·8	48·0	5·0	4·6		4·5
37·3	42·2	5·0	5·5		5·3
42·5	37·2	5·0	4·7		4·8
47·7	32·1	5·0	5·0		4·8
45·5	34·1	...	...		...
40·6	39·1	5·0	5·3		4·6
35·6	44·1	5·0	4·4		4·7
30·7	49·1	5·0	5·3		4·6
25·4	54·4	5·0	4·7		5·0
20·0	59·8	5·0	5·3		5·0

HOLMES' LEVEL No. 6

Mean Value of 1 Division of Scale = 0″·8635.

Reading of the Micrometer of the Bubble Tester	Bubble Reading		Value of the Arc in Seconds			
	Right Hand	Left Hand	As given by the Bubble Tester	As given by the Diagram of 1899	As given by the Diagram of 1900	As given by using a mean value
			″	″		″
80	78·0	5·2	...	...		...
70	65·0	18·1	10·0	9·7		11·2
50	38·1	45·0	20·0	22·6		23·2
30	16·9	66·7	20·0	22·1		18·5
30	16·3	75·1	...	...		...
35	22·3	69·5	5·0	6·5		5·0
40	27·4	64·2	5·0	5·0		4·5
45	33·0	58·8	5·0	5·6		4·8
50	37·9	53·7	5·0	5·7		4·3
55	43·0	48·7	5·0	5·9		4·4
53	40·4	51·1	...	...		...
48	35·8	55·9	5·0	4·4		4·1
43	31·2	60·4	5·0	5·8		3·9
38	25·1	66·7	5·0	6·3		5·4
33	19·8	71·9	5·0	4·7		4·5
28	13·5	78·2	5·0	5·8		5·4
50	51·6	40·2	...	...		...
60	63·8	28·0	10·0	10·0		10·5
35	34·0	57·7	25·0	27·0		25·7
25	23·7	68·1	10·0	11·3		8·9
40	38·6	53·0	...	...		...
60	63·8	27·9	20·0	22·3		21·7
50	51·1	40·6	10·0	10·4		11·0
30	29·2	62·3	20·0	21·8		18·8

H. M. COWIE.